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Asymptotic Properties and Numerical Results of Solutions for a System of Degenerate Parabolic Equations with Nonlinear Boundary Conditions

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Abstract. In this article, a system of parabolic equations with nonlinear boundary conditions is transformed into a system of self-similar (automodel) equations by applying a shapeshifting. An asymptotics for a compact supporting solution of the problem is proposed. The obtained asymptotic formulas are used as an initial approximation in the numerical solution of the problem. In numerical experiments $k = 3$ is taken, a finite-difference scheme is proposed, and a Python program is developed to perform computations for various parameter values, corresponding graphs obtained. Numerical experiments have shown that the iterative processes converge within 3 to 5 steps. The compactly supported self-similar asymptotic solution used as the initial approximation played an important role in ensuring the convergence of the iterative process. The graphs illustrate that, for different numerical parameter values, the reaction–diffusion processes occur at a finite speed.

Keywords: System of nonlinear equations, reaction-diffusion, global solution, asymptotic, numerical solution
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1. Introduction

The article considers the following system of nonlinear perturbed equations of parabolic type:

$$u_{it} = \left(|({u_i}^{m_i})_x|^{p_i-2}(u_i^{m_i})_x\right)_x, \quad i = 1, 2, \ldots, k, \quad x > 0, \quad 0 < t < T, \tag{1.1}$$

with nonlinear boundary conditions

$$-|({u_i}^{m_i})_x|^{p_i-2}(u_i^{m_i})_x(0,t) = \prod_{j=1}^{k} u_j^{q_{ij}}(0,t), \quad i = 1, 2, \ldots, k\,, \quad 0 < t < T, \tag{1.2}$$

the initial conditions

$$u_i(x, 0) = u_{i0}(x), \quad i = 1, 2, \ldots, k, \quad x > 0. \tag{1.3}$$

We study the self-similar asymptotics and numerical solutions of problem (1.1) - (1.3). Here $k \geq 1,\ p_i > \frac{1}{m_i} - 1,\ m_i \geq 1,\ q_{ij} > 0\ \ (i = 1, 2, ..., k)$ are continuous nonnegative functions on $\mathbb{R}_+$.

The system of nonlinear parabolic-type equations (1.1) represents a general mathematical model describing nonlinear heat conduction, diffusion, filtration, chemical reactions, and explosive processes, and is widely used in mathematical modeling of various natural phenomena [1]- [2].

It is known that in the region where $u_i = 0$, the system of equations (1.1) becomes degenerate and has no solution in the classical sense. In such cases, generalized solutions that possess the properties $|\ {u_{ix}}^{m_i}|^{p_i-2}(u_i^{m_i})_x \in C\,(Q)\,,\, 0 \leq u_i \in C\,(Q)$, are considered and this solution satisfies the system of equations (1.1) in the sense of an integral uniqueness [1].

The nonlinear parabolic equations (1.1) originates from the theory of turbulent diffusion [3],[4] (see also the references therein) and appears in population dynamics, chemical reactions, heat transfer, and other related fields. Equation (1.1) encompasses, as particular cases, the porous medium operator (when $p = 0$) and the gradient-diffusive p-Laplacian operator (when $m = 1$), both of which are subjects of intensive study ([5], [6], [7], [8], [9], [10]-[11], [12] and the references therein).

For nonlinear mathematical models, the problems of blow-up and global existence conditions, as well as the study of propagation speeds, have been extensively investigated ([1], [2]- [5], [6], [13], [7], [8]-[14], [15]-[16], [17] and the references therein). In particular, the critical Fujita exponents play an important role in the initial and boundary value problems for various nonlinear parabolic equations of mathematical physics ([5], [9], [12] and the references therein). The concept of Fujita critical exponents was introduced by Fujita in the 1960s during the study of the heat conduction problem with a nonlinear source term [13].

In the work of Galaktionov and Levine [4], the case of a single equation was studied in the following form:

$$\begin{cases} u_t = (u^m)_{xx}, & x > 0,\ 0 < t < T, \\ -(u^m)_x(0,t) = u^p(0,t), & 0 < t < T, \\ u(x,0) = u_0(x) & x > 0, \end{cases} \tag{1.4}$$

in addition, they also examined the heat conduction equation with gradient diffusion in the following form:

$$\begin{cases} u_t = \left(|u_x|^{m-1}u_x\right)_{x}, & x > 0,\ 0 < t < T, \\ -|u_x|^{m-1}u_x(0,t) = u^p(0,t), & 0 < t < T, \\ u(x,0) = u_0(x) & x > 0. \end{cases} \tag{1.5}$$

Here $m \geq 1,\ p > 0$ and u_0 is a compactly supported nonnegative function (i.e. defined in a bounded domain) serving as the initial function.

They showed that for problem (1.4), the critical exponent for global existence of the solution is $p_0 = \frac{1}{2}(m+1)$ and the critical Fujita exponent is $p_c = m+1$. Moreover, for problem (1.5), the critical exponent for global existence is $p_0 = \frac{2m}{m+1}$ and the critical Fujita exponent is $p_c = 2m$. The critical exponents for global existence and the Fujita critical exponent for problem (1.5) were studied in [7] for the special case $m = 1$.

Xiang, Chen, and Mu [11] investigated the following boundary value problem:

$$\begin{cases} u_{it} = \left(|u_{ix}|^{m_i-1}u_{ix}\right)_x & (i = 1,2,...,k),\ x > 0,\ 0 < t < T, \\ -|u_{ix}|^{m_i-1}u_{ix}(0,t) = u_{i+1}^{p_i}(0,t) & (i = 1,2,...,k),\ u_{k+1} := u_1,\ 0 < t < T, \\ u_i(x,0) = u_{i0}(x) & (i = 1,2,...,k),\ x > 0. \end{cases} \tag{1.6}$$

Here, the parameters satisfy $k \geq 2, m_i > 1, p_i > 0$ and the functions $u_{i0}, (i = 1,2,...,k)$ are continuous and nonnegative. They determined the critical curve of global existence and the Fujita-type critical curve. In article [18], Galaktionov and Levine studied the following equation:

$$u_t = \nabla\left(|\nabla u|^\sigma \nabla u^m\right) + u^p, \quad x \in R^N,\ \ t > 0, u(x,0) = u_0(x),\ \ x \in R^N,$$

here $\sigma > 0,\ m > 1,\ p > 1$ and $u_0(x)$ is a bounded, positive, and continuous function. They showed that the critical exponent is $p_c = m + \sigma + \frac{\sigma+2}{N}$.

Jiang and Zheng [14] considered the following problem:

$$\begin{cases} u_t = \left(|u_x|^\beta (u^m)_x\right)_x, & x > 0, 0 < t < T, \\ -|u_x|^\beta (u^m)_x(0,t) = u^p(0,t), & 0 < t < T, \\ u(x,0) = u_0(x), & x > 0, \end{cases} \tag{1.7}$$

here $m \geq 1,\ p > 0,\ \beta > 0$. They obtained the critical exponent for global existence $p_0 = \frac{2\beta+m+1}{\beta+2}$ and the critical Fujita exponent $p_c = 2\beta + m + 1$. These results represent a generalized form of the work by Galaktionov and Levine [4].

In [3], the following problem was considered:

$$\begin{cases} u_t = \left(|u_x|^{p_1} \left(u^{m_1}\right)_x\right)_x, & x > 0,\ 0 < t < T, \\ v_t = \left(|v_x|^{p_2} \left(v^{m_2}\right)_x\right)_x, & x > 0, 0 < t < T, \\ -|u_x|^{p_1} \left(u^{m_1}\right)_x(0,t) = \alpha_1(0,t)v^{\beta_2}(0,t), & 0 < t < T, \\ -|v_x|^{p_2} \left(v^{m_2}\right)_x(0,t) = \alpha_2(0,t)u^{\beta_1}(0,t), & 0 < t < T, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), & x > 0, \end{cases} \tag{1.8}$$

Here, the parameters satisfy $m_i \geq 1,\ p_i > 0,\ \alpha_i > 0,\ \beta_i > 0\ (i = 1,2)$ and the functions $u_0, v_0-$are nonnegative continuous functions with compact support on R_+. They found the global existence critical curve of and the Fujita-type critical curve; however, the classification of the existence and nonexistence of global solutions of the system (1.8) remains a very complex problem.

Using the sources mentioned above, the aim of this article is to find the asymptotic behavior of the solutions to problems (1.1)–(1.3) and based on the asymptotics obtained, to carry out a computational experiment and obtain visualization results for $k = 3$.

2. RESEARCH METHODOLOGY

To construct a self-similar system of equations, we seek the solution of the system (1.1) in the following form:

$$\bar{u}_i(x,t) = (\tau + t)^{-k_i} F_i(\xi_i), \quad \xi_i = x(\tau + t)^{-l_i}, \tag{2.1}$$

here $k_i + 1 = (k_i m_i + l_i)(p_i + 1) + l_i, \quad (k_i m_i + 1)(p_i + 1) = \sum\limits_{j=1}^{k} k_j q_{ij} \quad (i = 1, 2, ..., k)$.

As a result, of the calculations performed, problem (1.1)–(1.3) takes the following form:

$$\begin{cases} \left(\left|(F_i^{m_i})'\right|^{p_i}(F_i^{m_i})'\right)'(\xi_i) + k_i F_i(\xi_i) + l_i \xi_i F_i'(\xi_i) = 0, \\ -\left|(F_i^{m_i})'\right|^{p_i}(F_i^{m_i})'(0) = \prod\limits_{j=1}^{k} F_j^{q_{ij}}(0), \qquad i = 1, 2, ..., k. \end{cases} \tag{2.2}$$

For problem (2.2), we seek a self-similar solution in the following form:

$$\bar{F}_i(\xi_i) = \left(a_i - b_i \xi_i^{\frac{p_i+2}{p_i+1}}\right)_+^{\frac{p_i+1}{m_i(p_i+1)-1}},$$

here $a_i > 0$, $b_i = \frac{m_i(p_i+1)-1}{(p_i+2)m_i} k_i^{\frac{1}{p_i+1}} > 0,\ \ y_+ = \max(0, y)$.

Theorem 2.1. *For the solution of problem (2.2) with a compact support, as* $\xi_i \to \left(\frac{a_i}{b_i}\right)^{\frac{p_i+1}{p_i+2}}$ *the following asymptotic relation holds*

$$F_i(\xi_i) = \bar{F}_i\left(1 + o(1)\right). \tag{2.3}$$

Proof: We seek the solution of problem (2.2) in the following form:

$$F_i(\xi_i) = \bar{F}_i(\xi_i) w_i(\tau_i) \tag{2.4}$$

here $\tau_i = -\ln\left(a_i - b_i \xi_i^{\frac{p_i+2}{p_i+1}}\right)$ and $w_i(\tau_i)-$ are non-negative and bounded functions. Substituting solution (2.4) into problem (2.2), we obtain the following expression:

$$\frac{dF_i^{m_i}}{d\xi_i} = \frac{d\bar{F}_i^{m_i}}{d\tau_i}\frac{d\tau_i}{d\xi_i} w_i^{m_i}(\tau_i) + \bar{F}_i^{m_i}(\xi_i)\frac{dw_i^{m_i}(\tau_i)}{d\tau_i}\frac{d\tau_i}{d\xi_i} = \frac{d\tau_i}{d\xi_i}\left[\frac{d\bar{F}_i^{m_i}}{d\tau_i} w_i(\tau_i) + \bar{F}_i^{m_i}(\xi_i)\frac{dw_i^{m_i}(\tau_i)}{d\tau_i}\right] =$$

$$= \left| \begin{array}{l} \bar{F}_i^{m_i} = e^{-\tau_i \cdot \frac{(p_i+1)m_i}{m_i(p_i+1)-1}} \\ \frac{d\bar{F}_i^{m_i}}{d\tau_i} = \frac{(p_i+1)m_i}{m_i(p_i+1)-1} e^{-\tau_i \cdot \frac{(p_i+1)m_i}{m_i(p_i+1)-1}} \end{array} \right| = \left| \begin{array}{l} \tau_i = -\ln(a_i - b_i \xi_i^{\frac{p_i+2}{p_i+1}}) \\ \frac{d\tau_i}{d\xi_i} = -\frac{1}{a_i - b_i \xi_i^{\frac{p_i+2}{p_i+1}}} \left(-b_i \frac{p_i+2}{p_i+1} \xi_i^{\frac{1}{p_i+1}} \right) = \\ = \frac{b_i \xi_i^{\frac{1}{p_i+1}}}{a_i - b_i \xi_i^{\frac{p_i+2}{p_i+1}}} \frac{p_i+2}{p_i+1} = \frac{b_i}{e^{-\tau_i}} \frac{p_i+2}{p_i+1} \xi_i^{\frac{1}{p_i+1}}; \end{array} \right| =$$

$$= \frac{b_i(p_i+2)e^{\tau_i}}{p_i+1} \xi_i^{\frac{1}{p_i+1}} \left[-\frac{(p_i+1)m_i}{m_i(p_i+1)-1} e^{-\tau_i \frac{(p_i+1)m_i}{m_i(p_i+1)-1}} w_i^{m_i}(\tau_i) + e^{-\tau_i \frac{(p_i+1)m_i}{m_i(p_i+1)-1}} \left(w_i^{m_i}(\tau_i) \right)' \right] =$$

$$= -b_i m_i (p_i+2) \xi_i^{\frac{1}{p_i+1}} e^{-\tau_i \frac{1}{m_i(p_i+1)-1}} \left[\frac{\left(w_i^{m_i}(\tau_i) \right)'}{(p_i+1)m_i} - \frac{w_i^{m_i}(\tau_i)}{m_i(p_i+1)-1} \right],$$

$\frac{d}{d\xi_i} = \frac{d}{d\tau_i} \frac{d\tau_i}{d\xi_i} = -\frac{b_i(p_i+2)e^{\tau_i}}{(p_i+1)} \xi_i^{\frac{1}{p_i+1}} \frac{d}{d\tau_i}$.

Through the found equalities, we obtain the following:

$$\frac{b_i(p_i+2)e^{\tau_i}}{p_i+1} \xi_i^{\frac{1}{p_i+1}} \frac{d}{d\tau_i} \left[(m_i b_i (p_i+2))^{p_i+1} \xi_i e^{-\tau_i \frac{p_i+1}{m_i(p_i+1)-1}} \left[\frac{\left(w_i^{m_i}(\tau_i) \right)'}{(p_i+1)m_i} - \frac{w_i^{m_i}(\tau_i)}{m_i(p_i+1)-1} \right]^{p_i+1} \right] +$$

$$+ k_i e^{-\tau_i \frac{p_i+1}{m_i(p_i+1)-1}} w_i(\tau_i) + l_i b_i (p_i+2) \xi_i^{\frac{p_i+2}{p_i+1}} e^{-\tau_i \frac{m_i(p_i+1)-2-p_i}{m_i(p_i+1)-1}} \left[\frac{\left(w_i(\tau_i) \right)'}{(p_i+1)} - \frac{w_i(\tau_i)}{m_i(p_i+1)-1} \right] = 0. \quad (2.5)$$

Let us introduce the following notation:

$$L_1^{m_i}(w_i) = \frac{\left(w_i^{m_i}(\tau_i) \right)'}{(p_i+1)m_i} - \frac{w_i^{m_i}(\tau_i)}{m_i(p_i+1)-1},$$

$$L_2(w_i) = \frac{\left(w_i(\tau_i) \right)'}{(p_i+1)} - \frac{w_i(\tau_i)}{m_i(p_i+1)-1},$$

as a result

$$-\frac{b_i^{p_i+2}(p_i+2) m_i^{p_i+1} e^{\tau_i}}{p_i+1} \left(a_i - e^{-\tau_i} \right)^{\frac{1}{p_i+2}} b_i^{-1} \frac{d}{d\tau_i} \left[e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i} \left(a_i - e^{-\tau_i} \right)^{\frac{p_i+1}{p_i+2}} \left[L_1^{m_i}(w_i) \right]^{p_i+1} \right] +$$

$$+ k_i e^{-\tau_i \frac{p_i+1}{m_i(p_i+1)-1}} w_i(\tau_i) + l_i (p_i+2) \left(a_i - e^{-\tau_i} \right) e^{-\tau_i \frac{m_i(p_i+1)-2-p_i}{m_i(p_i+1)-1}} L_2(w_i) = 0. \quad (2.6)$$

From (2.6), we calculate $\frac{d}{d\tau_i}$:

$$\frac{d}{d\tau_i} \left[e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i} \left(a_i - e^{-\tau_i} \right)^{\frac{p_i+1}{p_i+2}} \right] = e^{-\tau_i} \frac{p_i+1}{p_i+2} \left(a_i - e^{-\tau_i} \right)^{-\frac{1}{p_i+2}} e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i} -$$

$$- \frac{p_i+1}{m_i(p_i+1)-1} \left(a_i - e^{-\tau_i} \right)^{\frac{p_i+1}{p_i+2}} e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i} =$$

$$= e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i} \left(a_i - e^{-\tau_i} \right)^{\frac{p_i+1}{p_i+2}} \times \left[\frac{p_i+1}{p_i+2} e^{-\tau_i} \left(a_i - e^{-\tau_i} \right)^{-1} - \frac{p_i+1}{m_i(p_i+1)-1} \right],$$

$$\frac{d}{d\tau_i} \left[e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i} \left(a_i - e^{-\tau_i} \right)^{\frac{p_i+1}{p_i+2}} \left[L_1^{m_i}(w_i) \right]^{p_i+1} \right] = e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i} \left(a_i - e^{-\tau_i} \right)^{\frac{p_i+1}{p_i+2}} \times$$

$$\times \left[\frac{p_i+1}{p_i+2} e^{-\tau_i} \left(a_i - e^{-\tau_i} \right)^{-1} - \frac{p_i+1}{m_i(p_i+1)-1} \right] \left[L_1^{m_i}(w_i) \right]^{p_i+1} +$$

$$e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i}\left(a_i-e^{-\tau_i}\right)^{\frac{p_i+1}{p_i+2}}\frac{d}{d\tau_i}(L_i^{m_i}(\tau_i))^{p_i+1}=e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i}\left(a_i-e^{-\tau_i}\right)^{\frac{p_i+1}{p_i+2}}\times$$
$$\times\left[\left[\frac{p_i+1}{p_i+2}e^{-\tau_i}\left(a_i-e^{-\tau_i}\right)^{-1}-\frac{p_i+1}{m_i(p_i+1)-1}\right][L_1{}^{m_i}(w_i)]^{p_i+1}+\frac{d}{d\tau_i}(L_i^{m_i}(\tau_i))^{p_i+1}\right]=0. \tag{2.7}$$

Substituting (2.7) into (2.6) we obtain :

$$\frac{b_i{}^{p_i+2}(p_i+2)^{p_i+2}m_i{}^{p_i+1}e^{\tau_i}}{p_i+1}\left(a_i-e^{-\tau_i}\right)^{\frac{1}{p_i+2}}b_i^{-1}\left[\left(a_i-e^{-\tau_i}\right)^{\frac{p_i+1}{p_i+2}}e^{-\frac{p_i+1}{m_i(p_i+1)-1}\tau_i}\times\right.$$
$$\left.\times\left[\left[\frac{p_i+1}{p_i+2}e^{-\tau_i}\left(a_i-e^{-\tau_i}\right)^{-1}-\frac{p_i+1}{m_i(p_i+1)-1}\right][L_1{}^{m_i}(w_i)]^{p_i+1}+\frac{d}{d\tau_i}(L_i^{m_i}(\tau_i))^{p_i+1}\right]\right]+$$
$$+k_ie^{-\tau_i\frac{p_i+1}{m_i(p_i+1)-1}}w_i(\tau_i)+l_i(p_i+2)\left(a_i-e^{-\tau_i}\right)e^{-\tau_i\frac{m_i(p_i+1)-2-p_i}{m_i(p_i+1)-1}}L_2(w_i)=0,$$
$$\left[\frac{p_i+1}{p_i+2}\frac{e^{-\tau_i}}{\left(a_i-e^{-\tau_i}\right)^{-1}}-\frac{p_i+1}{m_i(p_i+1)-1}\right][L_1{}^{m_i}(w_i)]^{p_i+1}+\frac{d}{d\tau_i}(L_i^{m_i}(\tau_i))^{p_i+1}+$$
$$+\frac{k_ie^{-\tau_i}b_i}{t_i\left(a_i-e^{-\tau_i}\right)}w_i(\tau_i)+\frac{l_ib_i(p_i+2)}{t_i}L_2(w_i)=0, \tag{2.8}$$

here $t_i=\frac{b_i{}^{p_i+2}(p_i+2)^{p_i+2}m_i{}^{p_i+1}}{p_i+1}$, $d_1=\frac{p_i+1}{p_i+2}$, $f_1(\tau_i)=\frac{e^{-\tau_i}}{a_i-e^{-\tau_i}}$, $d_2=\frac{k_ib_i}{t_i}$, $d_3=\frac{l_ib_i(p_i+2)}{t_i}$.

Equation (2.8) takes the following form:

$$\left[d_1f_1(\tau_i)-\frac{p_i+1}{m_i(p_i+1)-1}\right][L_1{}^{m_i}(w_i)]^{p_i+1}+\frac{d}{d\tau_i}(L_i^{m_i}(\tau_i))^{p_i+1}+d_2f_1(\tau_i)w_i(\tau_i)+d_3L_2(w_i)=0. \tag{2.9}$$

The study of the solutions of the last equation is equivalent to the study of the solutions of equation (2.2), and each of them satisfies the following inequalities on the interval $[\tau_0,+\infty)$

$$w_i{}^{m_i}(\tau_i)>0,\quad \frac{w_i{}^{m_i}(\tau_i)}{(p_i+1)m_i}-\frac{\left(w_i{}^{m_i}(\tau_i)\right)'}{m_i(p_i+1)-1}\neq 0.$$

Examine whether the solution $w_i{}^{m_i}(\tau_i)$ of equation (2.2) is bounded and has a limit w_0 or becomes unbounded as $\eta\to+\infty$. Suppose that

$$\nu_i(\tau_i)=(L_1{}^{m_i}(w_i))^{p_i+1}.$$

Then, for the derivative of the function $\nu_i(\tau_i)$, we obtain the following:

$$\nu'_i=\left[d_1f_1(\tau_i)-\frac{p_i+1}{m_i(p_i+1)-1}\right]\nu_i+d_2f_1(\tau_i)w_i(\tau_i)+d_3L_2(w_i).$$

To analyze the solutions of the last equation, we introduce an auxiliary function:

$$\theta_i(\tau_i,\mu_i)=\left[d_1f_1(\tau_i)-\frac{p_i+1}{m_i(p_i+1)-1}\right]\mu_i+d_2f_1(\tau_i)w_i(\tau_i)+d_3L_2(w_i), \tag{2.10}$$

here $\mu_i-$is a real number. It follows that each function $\theta_i(\tau_i,\mu_i)$is defined in the interval $[\tau_1,+\infty)$ and within this interval, one of the following inequalities holds:

$$\nu'_i(\tau_i)>0,\quad \nu'_i(\tau_i)<0.$$

Therefore, we analyze equation (2.10) taking into account Bol's theorem. It follows that for the function $\nu_i(\tau_i)$ a limit exists for $\tau_i\in[\tau_1,+\infty)$.

Now, let us consider the limiting case. It can be seen in the following:

$$\xi_i\to\left(\frac{a_i}{b_i}\right)^{\frac{p_i+1}{p_i+2}},\ \lim_{\tau_i\to+\infty}f_1(\tau_i)\to 0\quad .$$

Thus, from the existence of the above limit and the facts that $(w_i{}^{m_i})' = 0, \quad (w_i)' = 0$ it follows that

$$\left(d_1 f_1(\tau_i) - \frac{p_i+1}{m_i(p_i+1)-1}\right)\left[\frac{w_i{}^{m_i}(\tau_i)}{(p_i+1)m_i} - \frac{(w_i{}^{m_i}(\tau_i))'}{m_i(p_i+1)-1}\right]^{p_i+1} + \frac{d}{d\tau_i}(L_1{}^{m_i}(w_i))^{p_i+1} +$$

$$+ d_2 f_1(\tau_i) w_i(\tau_i) + d_3\left[\frac{(w_i(\tau_i))'}{p_i+1} - \frac{w_i(\tau_i)}{m_i(p_i+1)-1}\right] = 0,$$

$$-\frac{p_i+1}{m_i(p_i+1)-1}\left[\frac{w_i{}^{m_i}(\tau_i)}{(p_i+1)m_i} - \frac{(w_i{}^{m_i}(\tau_i))'}{m_i(p_i+1)-1}\right]^{p_i+1} + d_3\left[\frac{(w_i(\tau_i))'}{p_i+1} - \frac{w_i(\tau_i)}{m_i(p_i+1)-1}\right] = 0.$$

In this case, the following system of algebraic equations with respect to w_i arises:

$$-\frac{p_i+1}{m_i(p_i+1)-1}\left(\frac{w_i(\tau_i)}{m_i(p_i+1)-1}\right)^{p_i+1} = d_2\frac{w_i(\tau_i)}{m_i(p_i+1)-1}.$$

From this system of equations it follows that $w_i = 1$. According to (2.3), $F_i(\xi_i) = \bar{F}_i(1 + o(1))$ the theorem is proved.

3. Computational Experiment and Visualization Results

First, we construct a numerical scheme to solve problem (1.1)–(1.3). In the domain $Q = \{(t,x) : 0 \le t < T,\ 0 \le x \le b\}$ we construct a grid in time and space as follows:

$V_\tau = \{t_j = j\tau, \tau > 0, j = 1,2,...,m;\ m\tau = T,\ T > 0\}$, $S_h = \{x_i = ih; h > 0; i = 1,2,...,n; nh = b\}$ here $\tau = \frac{T}{m}$, and $h = \frac{b}{n}$. To solve problem (1.1)–(1.3) numerically for $k = 3$, we use an implicit scheme.

$$\begin{cases} \dfrac{y_i^{j+1} - y_i^j}{\tau} = \dfrac{1}{h^2}\left[D_{i+1}^j(y_{i+1}{}^j)\left(y_{i+1}^{j+1} - y_i^{j+1}\right) - D_i^j(y_i{}^j)\left(y_i^{j+1} - y_{i-1}^{j+1}\right)\right], \\ \dfrac{z_i^{j+1} - z_i^j}{\tau} = \dfrac{1}{h^2}\left[M_{i+1}^j(z_{i+1}^j)\left(z_{i+1}^{j+1} - z_i^{j+1}\right) - M_i^j(z_i^j)\left(z_i^{j+1} - z_{i-1}^{j+1}\right)\right], \\ \dfrac{t_i^{j+1} - t_i^j}{\tau} = \dfrac{1}{h^2}\left[N_{i+1}^j(t_{i+1}^j)\left(t_{i+1}^{j+1} - t_i^{j+1}\right) - N_i^j(t_i^j)\left(t_i^{j+1} - t_{i-1}^{j+1}\right)\right], \end{cases} \tag{3.1}$$

$$i = 2,...,n-1, j = 0,1,...,m-1,$$

$$y_i^0 = y(x_i,0),\ z_i^0 = z(x_i,0),\ t_i^0 = t(x_i,0),$$
$$i = 0,1,...,n,$$

$$\begin{cases} -D_1(y_1)\cdot\dfrac{\left(y_1^{j+1}\right)^{m_0} - \left(y_0^{j+1}\right)^{m_0}}{h} = (u_j)^{q_{1j}}, \\ -M_1(z_1)\cdot\dfrac{\left(z_1^{j+1}\right)^{m_0} - \left(z_0^{j+1}\right)^{m_0}}{h} = (u_j)^{q_{2j}}, \\ -N_1(t_1)\cdot\dfrac{\left(t_1^{j+1}\right)^{m_0} - \left(t_0^{j+1}\right)^{m_0}}{h} = (u_j)^{q_{3j}}, \end{cases} \tag{3.2}$$

$$\begin{cases} y_n^j = \varphi_1(k_j), \\ z_n^j = \varphi_2(k_j), \quad j = 2,3,...,m \\ t_n^j = \varphi_3(k_j) \end{cases} \tag{3.3}$$

a)

$$\begin{cases} D_i^j(y_i^j) = D_i^j\left(\dfrac{y_i^j - y_{i-1}^j}{2}\right), \\ M_i^j(z_i^j) = M_i^j\left(\dfrac{z_i^j - z_{i-1}^j}{2}\right), \\ N_i^j(t_i^j) = N_i^j\left(\dfrac{t_i^j - t_{i-1}^j}{2}\right), \end{cases} \tag{3.4}$$

b)

$$\begin{cases} D_i^j\left(y_i^j\right) = \dfrac{D_i^j(y_i^j) - D_{i-1}^j(y_{i-1}^j)}{2}, \\ M_i^j\left(z_i^j\right) = \dfrac{M_i^j(z_i^j) - M_{i-1}^j(z_{i-1}^j)}{2}, \\ N_i^j\left(t_i^j\right) = \dfrac{N_i^j(t_i^j) - N_{i-1}^j(t_{i-1}^j)}{2}, \end{cases} \tag{3.5}$$

here, equations (3.4)–(3.5) are written using the Samarskii–Sobol scheme. If we apply formula (3.4), we obtain

$$\begin{cases} D_i^j(y_i^j) = m_i\dfrac{\left(y_i^j\right)^{m_i-1} + \left(y_{i-1}^j\right)^{m_i-1}}{2}\left|\dfrac{\left(y_i^j\right)^{m_i} - \left(y_{i-1}^j\right)^{m_i}}{h}\right|^{p_1}, \\ D_{i+1}^j(y_{i+1}^j) = m_i\dfrac{\left(y_i^j\right)^{m_i-1} + \left(y_{i+1}^j\right)^{m_i-1}}{2}\left|\dfrac{\left(y_i^j\right)^{m_i} - \left(y_{i-1}^j\right)^{m_i}}{h}\right|^{p_1}, \end{cases}$$

$$\begin{cases} M_i^j(z_i^j) = m_i\dfrac{\left(z_i^j\right)^{m_i-1} + \left(z_{i-1}^j\right)^{m_i-1}}{2}\left|\dfrac{\left(z_i^j\right)^{m_i} - \left(z_{i-1}^j\right)^{m_i}}{h}\right|^{p_2}, \\ M_{i+1}^j(z_{i+1}^j) = m_i\dfrac{\left(z_i^j\right)^{m_i-1} + \left(z_{i+1}^j\right)^{m_i-1}}{2}\left|\dfrac{\left(z_{i+1}^j\right)^{m_i} - \left(z_i^j\right)^{m_i}}{h}\right|^{p_2}, \end{cases} \tag{3.6}$$

$$\begin{cases} N_i^j(t_i^j) = m_i\dfrac{\left(t_i^j\right)^{m_i-1} + \left(t_{i-1}^j\right)^{m_i-1}}{2}\left|\dfrac{\left(t_i^j\right)^{m_i} - \left(t_{i-1}^j\right)^{m_i}}{h}\right|^{p_3}, \\ N_{i+1}^j(t_{i+1}^j) = m_i\dfrac{\left(t_i^j\right)^{m_i-1} + \left(t_{i+1}^j\right)^{m_i-1}}{2}\left|\dfrac{\left(t_{i+1}^j\right)^{m_i} - \left(t_i^j\right)^{m_i}}{h}\right|^{p_3}, \end{cases}$$

if we apply formula (3.6), we obtain:

$$\begin{cases} D_i^j(y_i^j) = m_i\dfrac{\left(y_i^j\right)^{m_i-1} + \left(y_{i-1}^j\right)^{m_i-1}}{2}\,\dfrac{\left|\frac{y_i^j - y_{i-1}^j}{h}\right|^{p_1} + \left|\frac{y_{i-1}^j - y_{i-2}^j}{h}\right|^{p_1}}{2}, \\ D_{i+1}^j(y_{i+1}^j) = m_i\dfrac{\left(y_i^j\right)^{m_i-1} + \left(y_{i+1}^j\right)^{m_i-1}}{2}\,\dfrac{\left|\frac{y_{i+1}^j - y_i^j}{h}\right|^{p_1} + \left|\frac{y_i^j - y_{i-1}^j}{h}\right|^{p_1}}{2}, \end{cases}$$

$$\begin{cases} M_i^j(z_i^j) = m_i \dfrac{\left(z_i^j\right)^{m_i-1} + \left(z_{i-1}^j\right)^{m_i-1}}{2} \dfrac{\left|\frac{z_i^j - z_{i-1}^j}{h}\right|^{p_2} + \left|\frac{z_{i-1}^j - z_{i-2}^j}{h}\right|^{p_2}}{2}, \\ M_{i+1}^j(z_{i+1}^j) = m_i \dfrac{\left(z_i^j\right)^{m_i-1} + \left(z_{i+1}^j\right)^{m_i-1}}{2} \dfrac{\left|\frac{z_{i+1}^j - z_i^j}{h}\right|^{p_2} + \left|\frac{z_i^j - z_{i-1}^j}{h}\right|^{p_2}}{2}, \end{cases} \tag{3.7}$$

$$\begin{cases} N_i^j(t_i^j) = m_i \dfrac{\left(t_i^j\right)^{m_i-1} + \left(t_{i-1}^j\right)^{m_i-1}}{2} \dfrac{\left|\frac{t_i^j - t_{i-1}^j}{h}\right|^{p_3} + \left|\frac{t_{i-1}^j - t_{i-2}^j}{h}\right|^{p_3}}{2}, \\ N_{i+1}^j(z_{i+1}^j) = m_i \dfrac{\left(t_i^j\right)^{m_i-1} + \left(t_{i+1}^j\right)^{m_i-1}}{2} \dfrac{\left|\frac{t_{i+1}^j - t_i^j}{h}\right|^{p_3} + \left|\frac{t_i^j - t_{i-1}^j}{h}\right|^{p_3}}{2}, \end{cases}$$

system (3.1) is a system of nonlinear equations with respect to y^{j+1}, z^{j+1}, and t^{j+1}. To obtain numerical solutions of this system, we use the simple iteration method to linearize it.

$$\begin{cases} \dfrac{\overset{s+1}{y}{}_i^{j+1} - \overset{s}{y}{}_i^j}{\tau} = \dfrac{1}{h^2}\left[D_{i+1}^j(y_{i+1}^j)\left(\overset{s+1}{y}{}_{i+1}^{j+1} - \overset{s+1}{y}{}_i^{j+1}\right) - D_i^j(y_i^j)\left(\overset{s+1}{y}{}_i^{j+1} - \overset{s+1}{y}{}_{i-1}^{j+1}\right)\right], \\ \dfrac{\overset{s+1}{z}{}_i^{j+1} - \overset{s}{z}{}_i^j}{\tau} = \dfrac{1}{h^2}\left[M_{i+1}^j(z_{i+1}^j)\left(\overset{s+1}{z}{}_{i+1}^{j+1} - \overset{s+1}{z}{}_i^{j+1}\right) - M_i^j(z_i^j)\left(\overset{s+1}{z}{}_i^{j+1} - \overset{s+1}{z}{}_{i-1}^{j+1}\right)\right], \\ \dfrac{\overset{s+1}{t}{}_i^{j+1} - \overset{s}{t}{}_i^j}{\tau} = \dfrac{1}{h^2}\left[N_{i+1}^j(t_{i+1}^j)\left(\overset{s+1}{t}{}_{i+1}^{j+1} - \overset{s+1}{t}{}_i^{j+1}\right) - N_i^j(t_i^j)\left(\overset{s+1}{t}{}_i^{j+1} - \overset{s+1}{z}{}_{i-1}^{j+1}\right)\right], \end{cases} \tag{3.8}$$

here, $s = 0, 1, 2, ...$

The performance of the iterative method is highly dependent on the selection of a suitable initial approximation. A well-chosen approximation ensures rapid convergence to the solution while preserving the physical meaning of the problem. A fitting choice for this initial approximation is the asymptotic solution of the problem.

For each time step, the initial iterative values $\overset{s+1}{y}{}_i$, $\overset{s+1}{z}{}_i$, $\overset{s+1}{t}{}_i$ are taken from the previous time step: $\overset{0}{y}{}^{j+1} = y^j$, $\overset{0}{z}{}^{j+1} = z^j, \overset{0}{t}{}^{j+1} = t^j$. When using an iterative scheme for calculations, the precision of the iteration (repetition process) is determined.

$$\begin{cases} \max\limits_{0\le i\le n}\left|\overset{s+1}{y}{}_i - \overset{s}{y}{}_i\right| < \varepsilon, \\ \max\limits_{0\le i\le n}\left|\overset{s+1}{z}{}_i - \overset{s}{z}{}_i\right| < \varepsilon, \\ \max\limits_{0\le i\le n}\left|\overset{s+1}{t}{}_i - \overset{s}{t}{}_i\right| < \varepsilon. \end{cases} \tag{3.9}$$

Let us introduce the following notation: $\bar{y}_i = y_i^{j+1}$, $\bar{z}_i = z_i^{j+1}$, $\bar{t}_i = t_i^{j+1}$. Then the systems (3.8) can be written in the following form.

$$\begin{cases} \overset{s}{A}_{1i}\,\overset{s+1}{\bar{y}}{}_{i-1} - \overset{s}{C}_{1i}\,\overset{s+1}{\bar{y}}{}_i + \overset{s}{B}_{1i}\,\overset{s+1}{\bar{y}}{}_{i+1} = -\overset{s}{F}_{1i}\,, \\ \overset{s}{A}_{2i}\,\overset{s+1}{\bar{z}}{}_{i-1} - \overset{s}{C}_{2i}\,\overset{s+1}{\bar{z}}{}_i + \overset{s}{B}_{2i}\,\overset{s+1}{\bar{z}}{}_{i+1} = -\overset{s}{F}_{2i}\,, \\ \overset{s}{A}_{3i}\,\overset{s+1}{\bar{z}}{}_{i-1} - \overset{s}{C}_{3i}\,\overset{s+1}{\bar{z}}{}_i + \overset{s}{B}_{3i}\,\overset{s+1}{\bar{z}}{}_{i+1} = -\overset{s}{F}_{3i}\,, \end{cases} \tag{3.10}$$

here,

$$\overset{s}{A}{}_{1i} = \frac{\tau}{h^2} D_i^j(\overset{s}{y}{}_i^j), \overset{s}{A}{}_{2i} = \frac{\tau}{h^2} M_i^j(\overset{s}{z}{}_i^j), \overset{s}{A}{}_{3i} = \frac{\tau}{h^2} N_i^j(\overset{s}{t}{}_i^j),$$

$$\overset{s}{B}_{1i} = \frac{\tau}{h^2} D_{i+1}^j(\overset{s}{y}{}_{i+1}^j), \overset{s}{B}_{2i} = \frac{\tau}{h^2} M_{i+1}^j(\overset{s}{z}{}_{i+1}^j), \overset{s}{B}_{3i} = \frac{\tau}{h^2} N_{i+1}^j(\overset{s}{t}{}_{i+1}^j),$$

$$\overset{s}{C}_{1i} = \frac{\tau}{h^2}\left(D_i^j\left(\overset{s}{y}{}_i^j\right) + D_{i+1}^j\left(\overset{s}{y}{}_{i+1}^j\right)\right) + 1, \overset{s}{C}_{2i} = \frac{\tau}{h^2}\left(M_i^j\left(\overset{s}{z}{}_i^j\right) + M_{i+1}^j\left(\overset{s}{z}{}_{i+1}^j\right)\right) + 1,$$

$$\overset{s}{C}_{3i} = \frac{\tau}{h^2}\left(N_i^j\left(\overset{s}{t}{}_i^j\right) + N_{i+1}^j\left(\overset{s}{t}{}_{i+1}^j\right)\right) + 1,$$

$$\overset{s}{F}_{1i} = \overset{s}{y}{}_i^j, \overset{s}{F}_{2i} = \overset{s}{z}{}_i^j, \overset{s}{F}_{3i} = \overset{s}{t}{}_i^j.$$

The required values at the end of the interval $0 \le x \le b$ can be obtained using Milne's second order precision formula.

$$\begin{aligned} \left.\frac{\partial y}{\partial x}\right|_0 &\approx \frac{-y_2 + 4y_1 - 3y_0}{2h}, \quad \left.\frac{\partial y}{\partial x}\right|_n \approx \frac{3y_n - 4y_{n-1} + y_{n-2}}{2h}, \\ \left.\frac{\partial z}{\partial x}\right|_0 &\approx \frac{-z_2 + 4z_1 - 3z_0}{2h}, \quad \left.\frac{\partial z}{\partial x}\right|_n \approx \frac{3z_n - 4z_{n-1} + z_{n-2}}{2h}, \\ \left.\frac{\partial t}{\partial x}\right|_0 &\approx \frac{-t_2 + 4t_1 - 3t_0}{2h}, \quad \left.\frac{\partial t}{\partial x}\right|_n \approx \frac{3t_n - 4t_{n-1} + t_{n-2}}{2h}. \end{aligned} \tag{3.11}$$

To obtain the numerical solution of the linear algebraic equation system (3.10), we use the sweep method. According to the sweep method:

$$\begin{cases} \bar{y}_i = \alpha_{1i}\bar{y}_{i+1} + \beta_{1i}, \\ \bar{z}_i = \alpha_{2i}\bar{z}_{i+1} + \beta_{2i}, \\ \bar{t}_i = \alpha_{3i}\bar{t}_{i+1} + \beta_{3i}, \end{cases} \tag{3.12}$$

here, $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \beta_{1i}, \beta_{2i}, \beta_{3i}$ are the coefficients that are calculated using the following formulas:

$$\begin{cases} \alpha_{1,i+1} = \dfrac{B_{1i}}{C_{1i} - \alpha_{1i}A_{1i}} \\ \beta_{1,i+1} = \dfrac{A_{1i}\beta_{1i} + F_{1i}}{C_{1i} - \alpha_{1i}A_{1i}} \end{cases}$$

$$\begin{cases} \alpha_{2,i+1} = \dfrac{B_{2i}}{C_{2i} - \alpha_{2i}A_{2i}} \\ \beta_{2,i+1} = \dfrac{A_{2i}\beta_{2i} + F_{2i}}{C_{2i} - \alpha_{2i}A_{2i}} \end{cases}$$

$$\begin{cases} \alpha_{3,i+1} = \dfrac{B_{3i}}{C_{3i} - \alpha_{3i}A_{3i}} \\ \beta_{3,i+1} = \dfrac{A_{3i}\beta_{3i} + F_{3i}}{C_{3i} - \alpha_{3i}A_{3i}} \end{cases}$$

here, $i = 1, 2, ..., n$ and the values $\alpha_{10}, \alpha_{20}, \alpha_{30}, \beta_{10}, \beta_{20}, \beta_{30}$ are determined from the boundary conditions (3.12).

A computational experiment was carried out using the numerical scheme constructed above. We present some of its results. For calculations, the grid step was taken as $h = 0.05$ the number of nodes $N = 300$ and the iteration accuracy was set to $\varepsilon = 10^{-3}$. Formulas (2.1) and (2.3) were used as an initial approximation for the iterative process.

□

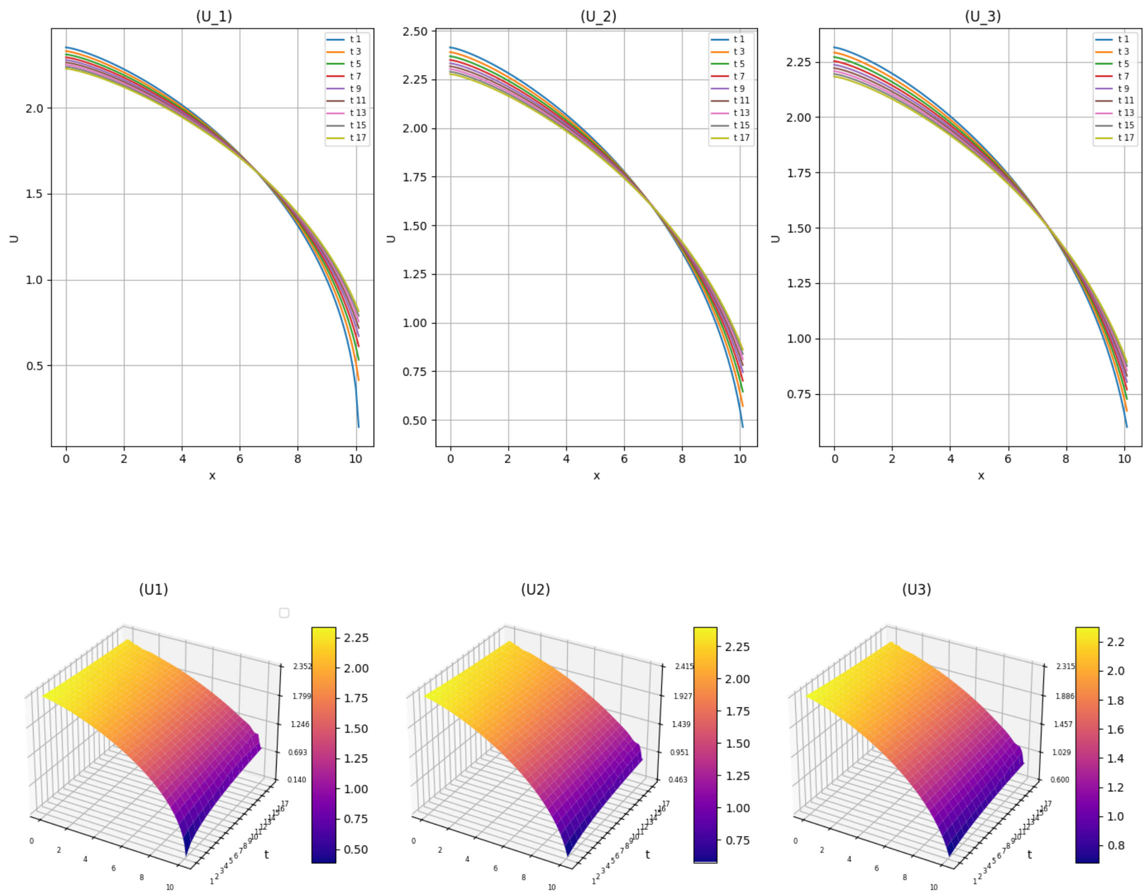

FIGURE 1. Numerical solution of problem (1.1) - (1.3).

q_{11}=4.61, q_{12}=4.21, q_{13}=4.42, q_{21}=4.32, q_{22}=4.51, q_{23}=4.61
q_{31}=4.33, q_{32}=4.63, q_{33}=4.83, p_1=2.18, p_2=2.2, p_3=2.21
m_1=2.59, m_2=2.52, m_3=2.63

Figure 1,2 shows the results of the numerical solution of the problem (1.1) - (1.3) for $p_i > \frac{1}{m_i} - 1$, corresponding to the case of slow diffusion. (2.1),(2.3) were taken as the initial approximation for the iterative process. At $p_i > \frac{1}{m_i} - 1$, as follows from the asymptotic formulasv(2.3) of the graphs, the heat propagation occurs at a finite rate. The penetration depth of a thermal wave depends on the time and the wavefront (the point at which $u_i(x,t)$ becomes zero) for each medium located at the final point: $x_{\phi_{u_i}} = \left(\frac{a_i}{b_i}\right)^{\frac{p_i+1}{p_i+2}} (\tau + t)^{l_i} < \infty$.

Numerical experiments have shown that iterative processes converge in 3 to 5 steps. The asymptotics of the compactly supported self-similar solutions, taken as the initial approximation, played an important role in ensuring the convergence of the iterative process. In the graphs presented above, it can be seen that for the given numerical parameter values, the reaction-diffusion processes occur at a finite speed.

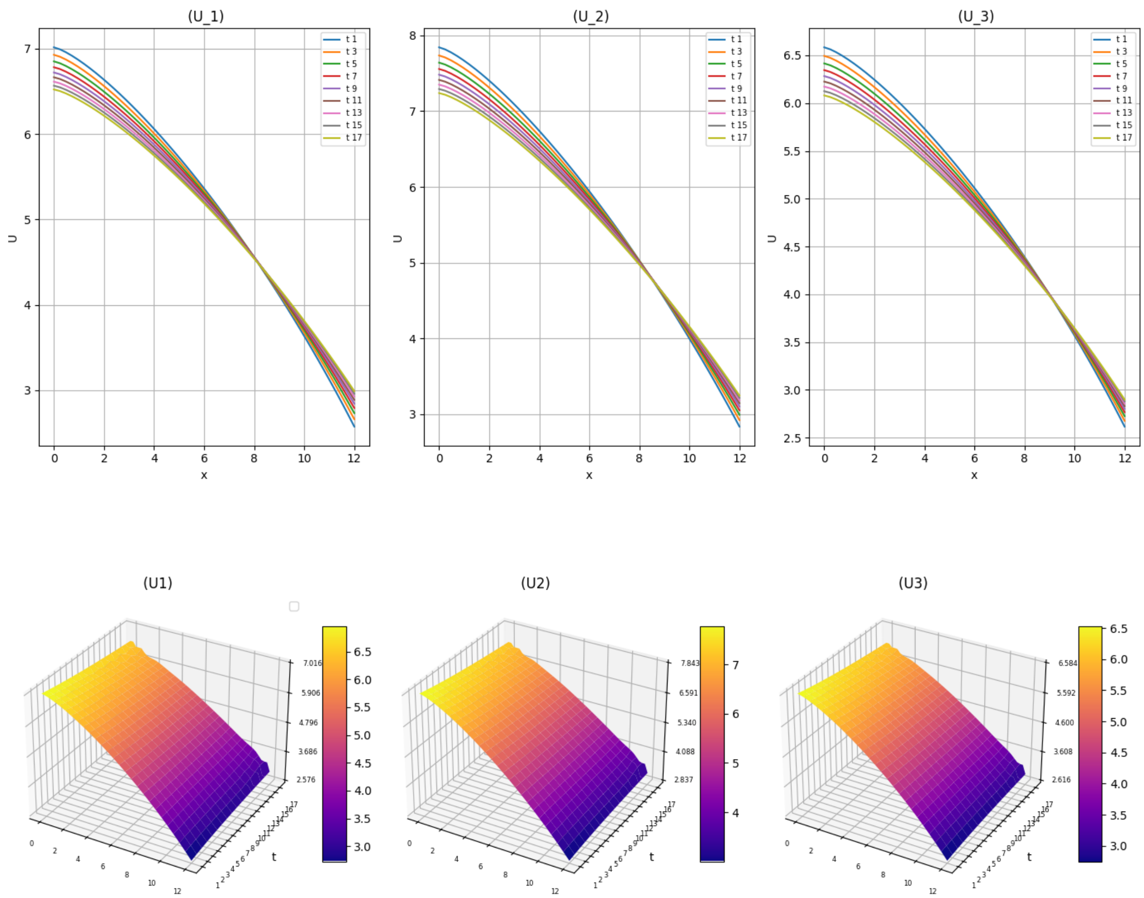

FIGURE 2. Numerical solution of problem (1.1) - (1.3).

q_{11}=3.601, q_{12}=3.201, q_{13}=3.402, q_{21}=3.302, q_{22}=3.501, q_{23}=3.601
q_{31}=3.303, q_{32}=3.603, q_{33}=3.803, p_1=2.18, p_2=2.19, p_3=2.21
m_1=1.59, m_2=1.52, m_3=1.63.

4. Conclusion

It is well known that in the numerical solution of nonlinear problems, one of the main issues is choosing an initial approximation that ensures rapid convergence of the iterative process to the exact solution and preserving the qualitative properties of nonlinear processes. This problem is solved by using the asymptotic formulas constructed above as the initial approximation, corresponding to the values of the numerical parameters. The results of several computational experiments for $k = 3$, along with the obtained graphs and the analysis of numerical solutions, are presented. The outcomes of the computational experiments demonstrated the efficiency of the proposed methods, while the numerical solutions reflected the nonlinear characteristics of the process under consideration.

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