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An evasion game for an infinite system of ternary differential equations in the Hilbert space l_2

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Abstract. In this paper, an evasion game for an infinite system of ternary differential equations is studied. The game is considered in the Hilbert space l_2. The control parameters of pursuer and evader are subject to geometric constraints. The purpose is to construct strategies for the evader to avoid being captured in the game. Equations for guaranteed evasion time for the evader are derived, describing how long the evader can avoid capture. Furthermore, this research paper contributes to understanding the concept of a guaranteed evasion game.

Keywords: Differential game, evader, control, strategy, infinite system of differential equations, geometric constraint
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1. Introduction

In the mid of 20th century, the theory of differential games, a fascinating intersection of control theory and game theory, were formally established by R.Isaacs [1], who first used the concept of "Differential Games" in his work. This work inspired numerous researchers including Pontryagin L.S. [2, 3], Krasovskii N.N. [4] and others who obtained the fundamental results on differential games.

In the last decades, on the other hand, the researchers such as Azamov, Blagodatskikh, Chikrii, Ibragimov, Mamadaliev, Petrov, Samatov have been obtaining interesting and important results (see [5]-[6], [7]-[8], [9]-[10], [11]-[12]). Azamov and Ruziboev [5] explored time-optimal control problems associated with PDEs, while Blagodatskikh and Petrov's investigations of group pursuit strategies and impulsive controls provided critical insights into strategies for dynamic systems ([13, 14] and [9]-[10]). However, Chikrii's work [15, 16] provided critical insights into linear differential games with integral constraints.

A great advancement in differential games involved reducing differential game problems modelled as PDE to infinite systems of differential equations, methods like Fourier analysis. We can see using such transformation in studies by Ibragimov et al. [17], [18]-[19], who addressed infinite-dimensional pursuit-evasion games in the Hilbert space l_2.

The works of Satimov and Tukhtasinov [20],[21] focused on pursuit and evasion games in systems described by parabolic type equations. In these games, the conditions that guarantees pursuit and evasion were obtained. Other studies, such as Mamadaliev et al. [7]-[8], explored differential pursuit games with integral constraints and impulse controls. In the paper by Ibragimov [22], an evasion differential game of many pursuers and one evader was considered. The evader has an advantage over pursuers in control resource. A sufficient condition of an evasion was found and an evasion strategy was constructed. Moreover, in the paper by Kuchkarov et al. [23], a simple motion differential game of many pursuers and one evader was studied on manifolds with Euclidean metric. All players have equal dynamic capabilities. The condition that ensures a pursuit can be completed in the game was obtained, if the condition is not satisfied, an evasion is possible.

The works [24, 20, 21] are devoted to a differential game described by a PDE of the form

$$\frac{\partial f}{\partial t} = Af + u - v, \quad Af = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial f}{\partial x_i}\right).$$

In these works, the given equation was reduced the equation to infinite system of differential equations

$$\dot{f}_k + \lambda_k f_k = u_k - v_k, \;\; k = 1, 2, ..., \tag{1.1}$$

where v_k and u_k are control parameters of evader and pursuer, respectively, $f_k, u_k, v_k \in \mathbb{R}$, and coefficients λ_k, $k = 1, 2, ...$, satisfy the condition $0 < \lambda_1 \leq \lambda_2 \leq ... \to \infty$.

In the papers [25], [26], the following evasion differential game for an infinite system of binary differential equations

$$\begin{aligned} &\dot{x}_i = -\lambda_i x_i + y_i + u_{i1} - v_{i1}, && x_i(0) = x_{i0}, \\ &\dot{y}_i = -\lambda_i y_i + u_{i2} - v_{i2}, && y_i(0) = y_{i0}, \quad i = 1, 2, ..., \end{aligned} \tag{1.2}$$

was studied in the Hilbert space l_2.

The present paper aims to contribute to an evolving field by investigating an evasion game for an infinite system of ternary differential equations within the Hilbert space l_2. In this game, the control parameters of the players are subject to geometric constraints. We develop a strategy for the evader that guarantees an evasion during the game. Additionally, we provide an equation to determine the guaranteed evasion time.

1.1. **Statement of problem.** It is important to highlight that the Hilbert space under consideration is l_2, which is a vector space:

$$l_2 = \left\{ \alpha = (\alpha_1, \alpha_2, ..., \alpha_n, ...) \mid \sum_{n=1}^{\infty} \alpha_n^2 < \infty \right\},$$

whose elements are sequences of real numbers.

Moreover, the inner product and norm are defined as follows:

$$(\alpha, \beta) = \sum_{n=1}^{\infty} \alpha_n \beta_n, \quad ||\alpha|| = \sqrt{(\alpha, \alpha)}.$$

We examine a controlled object given by the following infinite system of differential equations

$$\begin{aligned} &\dot{x}_i = -\lambda_i x_i + y_i + u_{i1} - v_{i1}, && x_i(0) = x_{i0}, \\ &\dot{y}_i = -\lambda_i y_i + z_i + u_{i2} - v_{i2}, && y_i(0) = y_{i0}, \\ &\dot{z}_i = -\lambda_i z_i + u_{i3} - v_{i3}, && z_i(0) = z_{i0} \quad i = 1, 2, 3, ..., \end{aligned} \tag{1.3}$$

in the Hilbert space l_2, where λ_i are given positive numbers, $x_0 = (x_{10}, x_{20}, x_{30}, \ldots) \in l_2$, $y_0 = (y_{10}, y_{20}, y_{30}, \ldots) \in l_2$, $z_0 = (z_{10}, z_{20}, z_{30}, \ldots) \in l_2$ are initial positions, $u = (u_{11}, u_{12}, u_{13}, u_{21}, u_{22}, u_{23}, \ldots)$ and $v = (v_{11}, v_{12}, v_{13}, v_{21}, v_{22}, v_{23}, \ldots)$ are the control parameters of pursuer and evader, respectively. We suppose that $0 \leq t \leq T$, where T is a given sufficiently large number, and that $\varphi_0 = (\varphi_{10}, \varphi_{20}, \varphi_{30}, ...) = (x_{10}, y_{10}, z_{10}, x_{20}, y_{20}, z_{20}, x_{30}, y_{30}, z_{30}, \ldots) \neq 0$.

Assume that ρ and σ, $\rho > \sigma$, are given positive numbers.

Definition 1.1. A function $u(t) = (u_1(t), u_2(t), u_3(t), \ldots)$, $t \in [0, T]$, with measurable coordinates $u_i(t) = (u_{i1}(t), u_{i2}(t), u_{i3}(t))$, $i = 1, 2, ...$, subject to the condition

$$\sum_{i=1}^{\infty} (u_{i1}^2(t) + u_{i2}^2(t) + u_{i3}^2(t)) \leq \rho^2, \quad 0 \leq t \leq T, \tag{1.4}$$

is referred to as the admissible control of the pursuer.

Definition 1.2. A function $v(t) = (v_1(t), v_2(t), v_3(t), \ldots)$, $t \in [0, T]$, with measurable coordinates $v_i(t) = (v_{i1}(t), v_{i2}(t), v_{i3}(t))$, $i = 1, 2, ...$, subject to the condition

$$\sum_{i=1}^{\infty} (v_{i1}^2(t) + v_{i2}^2(t) + v_{i3}^2(t)) \leq \sigma^2, \quad 0 \leq t \leq T, \tag{1.5}$$

is referred to as the admissible control of the evader.

We define $S(\sigma)$ as the set of all admissible controls.

Let

$$A_i = \begin{bmatrix} -\lambda_i & 1 & 0 \\ 0 & -\lambda_i & 1 \\ 0 & 0 & -\lambda_i \end{bmatrix}, \quad i = 1, 2, ...,$$

then, we have

$$e^{A_i t} = e^{-\lambda_i t} \begin{bmatrix} 1 & t & \frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}, \quad i = 1, 2, ... \tag{1.6}$$

Let $C(0, T; l_2)$ be the space of continuous functions $\varphi(\cdot)$ such that $\varphi(t) \in l_2$ for each $0 \leq t \leq T$. We present the following proposition which is proved in [6].

Proposition 1.1. If $w(\cdot) \in S(\sigma)$, and $\lambda_i \geq 0$, $i = 1, 2, ...$, then there exists a unique solution $\varphi(t) = (\varphi_1(t), \varphi_2(t), ...)$ of the infinite system of differential equations (1.3) in the space $C(0, T; l_2)$.

Of course, for $i = 1, 2, ...$, we have (see [6])

$$\varphi_i(t) = e^{A_i t}\varphi_{i0} + \int_0^t e^{A_i(t-s)}(u_i(s) - v_i(s))ds, \quad i = 1, 2, ...$$

By denoting $\varphi_i(t) = e^{A_i t}\psi_i(t)$, it is not difficult to see that $\varphi(t) = 0$ implies $\psi(t) = 0$, where

$$\psi(t) = (\psi_1(t), \psi_2(t), ...), \quad \psi_i(t) = \varphi_{i0} + \int_0^t e^{-A_i s}(u_i(s) - v_i(s))ds, \quad i = 1, 2, ... \tag{1.7}$$

Let $\varphi_i(t) = (x_i(t), y_i(t), z_i(t))$, $\varphi_{i0} = (x_{i0}, y_{i0}, z_{i0})$, $v_i = (v_{i1}, v_{i2}, v_{i3})$.

Definition 1.3. We call the function

$$(t, \varphi, \xi, \eta, u) \to V(t, \varphi, \xi, \eta, u), \quad V : \mathbb{R}_+ \times l_2 \times [0, \rho] \times [0, \sigma] \times l_2 \to l_2,$$

the strategy of the evader, if, for any admissible control of pursuer $u = u(t)$, $t \in [0, T]$, the system with $v = V(t, \varphi(t), \rho(t), \sigma(t), u(t))$, has a unique solution

$$\varphi(t) = (\varphi_1(t), \varphi_2(t), \ldots) = (x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t), \ldots), \quad i = 1, 2, ...$$

after substitution of the players' controls into it.

Definition 1.4. We say that the strategy V of evader guarantees evasion on the interval $[0, \tau(V))$ from the initial state $\varphi_0 \in l_2$ if $\varphi(t) \neq 0$, $t \in [0, \tau(V))$, for any pursuer's admissible control $u(t)$, $t \in [0, T]$. We call the number $\tau(V)$ a guaranteed evasion time.

The evader is interested in maximizing the guaranteed evasion time θ by choosing its strategy V but the pursuer tries to minimize θ by using its control $u = u(t)$, $0 \leq t \leq T$.

Problem 1. Construct an admissible control of the evader and find an equation for a guaranteed evasion time θ in game (1.3).

2. MAIN RESULT

In this section we present an admissible control for the evader and determine a guaranteed evasion time θ in game (1.3).

Theorem 2.1. For any initial state $\varphi_0 = (\varphi_{10}, \varphi_{20}, \cdots) \neq 0$, the number

$$\theta = \sup_i \theta_i,$$

is a guaranteed evasion time in game (1.2), where

$$\theta_i = \frac{1}{\lambda_i} \ln \frac{-(\rho - \sigma) + \sqrt{(\rho - \sigma)^2 + 2\lambda_i|\varphi_{i0}|(\rho + \sigma)}}{\rho + \sigma}$$

if $\lambda_i \geq 1$, and

$$\theta_i = \frac{1}{2} \ln \left(\frac{2|\varphi_{i0}|}{\rho + \sigma} + 1 \right)$$

if $0 < \lambda_i < 1$.

Proof. To prove theorem, we construct a strategy for the evader that ensures $\varphi(t) \neq 0$ on $[0, \theta']$ where θ' is an arbitrary time satisfying the condition $0 < \theta' < \theta$, while the pursuer can use an arbitrary control $u = u(t)$.

Indeed, we can find $j \in \{1, 2, \cdots\}$ such that $\theta' < \theta_j$ by definition of θ. Thus, our main goal is to show that in the game (1.3), an evasion is possible on $[0, \theta_j)$. Observe $\varphi_{j0} \neq 0$ since otherwise $\theta_j = 0$ contradicting positivity of θ_j.

We present the control for the evader as follows:

$$\begin{aligned} v_j(t) &= -\sigma \frac{1}{1+t+\frac{1}{2}t^2} \begin{bmatrix} 1 & t & \frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix} e_j, \\ v_i(t) &= 0, \ i = 1, 2, \cdots, \ i \neq j, \quad t \in [0, \theta_j). \end{aligned} \tag{2.1}$$

where $e_j = \frac{\varphi_{j0}}{|\varphi_{j0}|}$.

Lemma 2.1. Control (2.1) serves is the admissible control for the evader.

Proof. To prove the lemma, we use (1.5)

$$\begin{aligned} |v_j(t)| &= \left| -\sigma \frac{1}{1+t+\frac{1}{2}t^2} \begin{bmatrix} 1 & t & \frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} \right| = \frac{\sigma}{1+t+\frac{1}{2}t^2} \left\| \begin{bmatrix} e_{j1} + te_{j2} + \frac{1}{2}t^2 e_{j3} \\ e_{j2} + te_{j3} \\ e_{j3} \end{bmatrix} \right\| \\ &\leq \frac{\sigma}{1+t+\frac{1}{2}t^2} \left\| \begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} + \begin{bmatrix} te_{j2} \\ te_{j3} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}t^2 e_{j3} \\ 0 \\ 0 \end{bmatrix} \right\| \leq \frac{\sigma}{1+t+\frac{1}{2}t^2} \left(1 + t + \frac{1}{2}t^2\right) = \sigma. \end{aligned}$$

Consequently, control (2.1) is admissible. □

For any admissible control $u(t)$ of the pursuer, where time $t \in [0, \theta_j)$, we have

$$\begin{aligned} \psi_j(t) &= \varphi_{j0} + \int_0^t e^{-A_j s} u_j(s) ds - \int_0^t e^{-A_j s} v_j(s) ds \\ &= \varphi_{j0} + \int_0^t e^{-A_j s} u_j(s) ds + \sigma e_j \int_0^t \frac{1}{1+s+\frac{1}{2}t^2} e^{-\lambda_j s} ds. \end{aligned} \tag{2.2}$$

From the following inequality

$$|(e^{-A_j s} u_j(s), e_j)| \leq |e^{-A_j s} u_j(s)||e_j| = |(e^{-A_j s} u_j(s)| = e^{\lambda_j s} \left| \begin{bmatrix} 1 & -s & \frac{1}{2}s^2 \\ 0 & 1 & -s \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{j1} \\ u_{j2} \\ u_{j3} \end{bmatrix} \right| \leq \rho e^{\lambda_j s} \left(1 + s + \frac{1}{2}s^2\right)$$

we have

$$|(e^{-A_j s} u_j(s), e_j)| \leq \rho e^{\lambda_j s} \left(1 + s + \frac{1}{2}s^2\right). \tag{2.3}$$

Now, we multiple both sides of equation (2.2) by e_j and by using (2.3), we obtain

$$\begin{aligned} \psi_j(t) e_j &= |\varphi_{j0}| + \int_0^t \left(e^{-A_j s} u_j(s), e_j\right) ds + \sigma \int_0^t \frac{1}{1+s+\frac{1}{2}s^2} e^{\lambda_j s} ds \\ &\geq |\varphi_{j0}| - \rho \int_0^t \left(1 + s + \frac{1}{2}s^2\right) e^{\lambda_j s} ds + \sigma \int_0^t \frac{1}{1+s+\frac{1}{2}s^2} e^{\lambda_j s} ds \\ &= |\varphi_{j0}| - (\rho + \sigma) \int_0^t \left(1 + s + \frac{1}{2}s^2\right) e^{\lambda_j s} ds + \\ &+ \sigma \int_0^t \left[\left(1 + s + \frac{1}{2}s^2\right) e^{\lambda_j s} + \frac{1}{1+s+\frac{1}{2}s^2} e^{\lambda_j s}\right] ds. \end{aligned}$$

Hence, by the arithmetic mean-geometric mean inequality we obtain

$$\begin{aligned}\psi_j(t)e_j >&|\varphi_{j0}| - (\rho+\sigma)\int_0^t (1+s+\frac{1}{2}s^2)e^{\lambda_j s}ds + 2\sigma\int_0^t e^{\lambda_j s}ds\\ =&|\varphi_{j0}| - (\rho+\sigma)\int_0^t (s+\frac{1}{2}s^2)e^{\lambda_j s}ds - (\rho+\sigma)\int_0^t e^{\lambda_j s}ds + 2\sigma\int_0^t e^{\lambda_j s}ds\\ =&|\varphi_{j0}| - (\rho+\sigma)\int_0^t (s+\frac{1}{2}s^2)e^{\lambda_j s}ds - (\rho-\sigma)\int_0^t e^{\lambda_j s}ds.\end{aligned}$$

Then, by using inequality $s+\frac{1}{2}s^2 < e^s$ we have the following

$$\psi_j(t)e_j > |\varphi_{j0}| - (\rho+\sigma)\int_0^t e^{(1+\lambda_j)s}ds - (\rho-\sigma)\int_0^t e^{\lambda_j s}ds.$$

We now consider two cases according to the values of λ_j. The first case is when $\lambda_j \geq 1$ and the second one is $0 < \lambda_j < 1$.

In the first case, by the condition $\lambda_j \geq 1$ we have

$$\begin{aligned}\psi_j(t)e_j >&|\varphi_{j0}| - (\rho+\sigma)\int_0^t e^{(1+\lambda_j)s}ds - (\rho-\sigma)\int_0^t e^{\lambda_j s}ds\\ \geq&|\varphi_{j0}| - (\rho+\sigma)\int_0^t e^{2\lambda_j s}ds - (\rho-\sigma)\int_0^t e^{\lambda_j s}ds.\end{aligned}$$

Since $t \in [0, \theta_j]$,

$$\begin{aligned}\psi_j(t)e_j >&|\varphi_{j0}| - (\rho+\sigma)\int_0^{\theta_j} e^{2\lambda_j s}ds - (\rho-\sigma)\int_0^{\theta_j} e^{\lambda_j s}ds\\ =&|\varphi_{j0}| - (\rho+\sigma)\frac{e^{2\lambda_j\theta_j}-1}{2\lambda_j} - (\rho-\sigma)\frac{e^{\lambda_j\theta_j}-1}{\lambda_j}\\ =&-(\rho+\sigma)\frac{e^{2\lambda_j\theta_j}}{2\lambda_j} + \frac{\rho+\sigma}{2\lambda_j} + |\varphi_{j0}| - (\rho-\sigma)\frac{e^{\lambda_j\theta_j}}{\lambda_j} + \frac{(\rho-\sigma)}{\lambda_j}\\ =&-(\rho+\sigma)\frac{e^{2\lambda_j\theta_j}}{2\lambda_j} - (\rho-\sigma)\frac{e^{\lambda_j\theta_j}}{\lambda_j} + |\varphi_{j0}| + \frac{(3\rho-\sigma)}{2\lambda_j}\\ >&-(\rho+\sigma)\frac{e^{2\lambda_j\theta_j}}{2\lambda_j} - (\rho-\sigma)\frac{e^{\lambda_j\theta_j}}{\lambda_j} + |\varphi_{j0}|.\end{aligned}$$

By the value of θ_j, we have

$$\psi_j(t)e_j > -(\rho+\sigma)\frac{e^{2\lambda_j\theta_j}}{2\lambda_j} - (\rho-\sigma)\frac{e^{\lambda_j\theta_j}}{\lambda_j} + |\varphi_{j0}| = 0. \tag{2.4}$$

In the second case, where $0 < \lambda_j < 1$, we obtain

$$\begin{aligned}\psi_j(t)e_j >&|\varphi_{j0}| - (\rho+\sigma)\int_0^t e^{(1+\lambda_j)s}ds + 2\sigma\int_0^t e^{\lambda_j s}ds\\ >&|\varphi_{j0}| - (\rho+\sigma)\int_0^t e^{(1+\lambda_j)s}ds > |\varphi_{j0}| - (\rho+\sigma)\int_0^t e^{2s}ds\\ >&|\varphi_{j0}| - (\rho+\sigma)\int_0^{\theta_j} e^{2s}ds = |\varphi_{j0}| - (\rho+\sigma)\frac{e^{2\theta_j}-1}{2} = 0.\end{aligned}$$

Hence, in both cases, we obtained that $\psi_j(t)e_j > 0, 0 \leq t < \theta_j$ which implies $\psi_j(t) \neq 0$. As the result, $\varphi_j(t) \neq 0$ by the equation (1.7), thus, $\varphi(t) \neq 0$ for $t \in [0, \theta_j)$. In particular, $\varphi(t) \neq 0$ on the interval $[0, \theta']$. Proof of Theorem 2.1 is complete. □

3. CONCLUSIONS

In this research paper, we have focused on investigating the guaranteed evasion time within the context of the game described by equation (1.3), while an evasion game for an infinite system of binary differential equations in the Hilbert space l_2 was studied in the papers [25], [26]. The central focus was to construct a guaranteed evasion strategy for the evader. The control parameters of both players are subject to geometric constraints. We derived explicit formulas to calculate the guaranteed evasion time. Future research may focus on broadening these results to study more complex constraints, multi-player scenarios, further advancing the applicability of these methods to real-world problems and also finding optimal evasion strategy is an open problem.

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