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Eta quotients of level 20 and weight 1

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Abstract. We find all the eta quotients in the spaces $M_1\left(\Gamma_0(20), \left(\frac{d}{*}\right)\right)$ with $d = -4, -20$ of modular forms, and we determine their Fourier coefficients, where $\left(\frac{d}{*}\right)$ is the Legendre-Jacobi-Kronecker symbol in the group of Dirichlet characters modulo 20 with values in the rational field $\mathbb{Q}$.

Keywords: Eta quotients, Modular forms, Fourier coefficients of cusp forms, eta function
MSC (2020): 11F11, 11F20, 11F30

1. Introduction

Let $\mathbb{N}$, $\mathbb{N}_0$, $\mathbb{Z}$, and $\mathbb{C}$ denote the sets of positive integers, nonnegative integers, integers, and complex numbers, respectively. Let $N \in \mathbb{N}$, $k \in \mathbb{Z}$, and let χ be a Dirichlet character whose modulus divides N. We define the congruence subgroup $\Gamma_0(N)$ of level N by

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z},\ ad - bc = 1,\ c \equiv 0 \pmod N \right\}.$$

Let $M_k(\Gamma_0(N), \chi)$ denote the space of modular forms of weight k and character χ on the congruence subgroup $\Gamma_0(N)$. The subspaces of Eisenstein series and cusp forms are denoted by $E_k(\Gamma_0(N), \chi)$ and $S_k(\Gamma_0(N), \chi)$, respectively. It is well-known that

$$M_k(\Gamma_0(N), \chi) = E_k(\Gamma_0(N), \chi) \oplus S_k(\Gamma_0(N), \chi). \tag{1.1}$$

For instance, (see [1, p. 83]).

The Dedekind eta function $\eta(z)$ is a holomorphic function on the upper half-plane

$$\mathbb{H} = \{z \in \mathbb{C} \mid \mathrm{Im}(z) > 0\},$$

defined by

$$\eta(z) = e^{\pi i z/12} \prod_{n=1}^{\infty} \left(1 - e^{2\pi i n z}\right).$$

A product of the form

$$f(z) = \prod_{1 \le \delta \mid N} \eta^{r_\delta}(\delta z), \tag{1.2}$$

where $r_\delta \in \mathbb{Z}$, not all zero, is called an eta quotient.

Let χ and ψ be Dirichlet characters. For $n \in \mathbb{N}$, we define the generalized divisor sum $\sigma_{(\chi,\psi)}(n)$ by

$$\sigma_{(\chi,\psi)}(n) = \sum_{1 \le m \mid n} \chi(m)\psi(n/m). \tag{1.3}$$

If $n \notin \mathbb{N}$, we set $\sigma_{(\chi,\psi)}(n) = 0$.

Let χ_0 denote the principal character satisfies $\chi_0(m) = 1$ if and only if $(m, N) = 1$, otherwise, $\chi_0(m) = 0$. In particular, we observe that $\sigma_{(\chi_0,\chi_0)}(n)$ coincides with the classical divisor-counting function

$$\sigma_0(n) = \sum_{1 \le m \mid n} 1.$$

We now define two Dirichlet characters modulo 20 by

$$\chi_1(m) = \left(\frac{-4}{m}\right), \qquad \chi_2(m) = \left(\frac{-20}{m}\right), \quad \chi_3(m) = \left(\frac{-5}{m}\right), \qquad (m \in \mathbb{Z}),$$

where $\left(\frac{d}{\cdot}\right)$ denotes the Legendre-Jacobi-Kronecker symbol considered as a Dirichlet character modulo 20 with values in the field of rational numbers with conductor 4, 20 and 5 respectively.

The cusps of $\Gamma_0(N)$ can be represented by rational numbers $\frac{a}{c}$, where $a \in \mathbb{Z}$, $c \in \mathbb{N}$, $c \mid N$, and $\gcd(a,c) = 1$; (see [2, p. 200]).

For the group $\Gamma_0(20)$, a complete set of representatives of the cusps can be chosen as

$$\frac{1}{20}, \quad \frac{1}{10}, \quad \frac{1}{5}, \quad \frac{1}{4}, \quad \frac{1}{2}, \quad \frac{1}{1}. \tag{1.4}$$

Let $f(z)$ be an eta quotient as defined in (1.2). A formula for the order $v_{a/c}(f)$ of f at the cusp $\frac{a}{c}$ (see [2, p. 200]) is given by

$$v_{a/c}(f) = \frac{N}{24 \gcd(c^2, N)} \sum_{1 \le \delta \mid N} \frac{\gcd(\delta, c)^2 \, r_\delta}{\delta}. \tag{1.5}$$

It follows from the dimension formulae [1, Section 6.3] that the only nontrivial modular form spaces of level 20 with trivial cuspidal subspaces are

$$M_1(\Gamma_0(20), \chi_1) \quad \text{and} \quad M_1(\Gamma_0(20), \chi_2).$$

Moreover, we have that

$$\begin{aligned} \dim\big(M_1(\Gamma_0(20), \chi_1)\big) &= \dim\big(E_1(\Gamma_0(20), \chi_1)\big) = 2, \\ \dim\big(M_1(\Gamma_0(20), \chi_2)\big) &= \dim\big(E_1(\Gamma_0(20), \chi_2)\big) = 2. \end{aligned} \tag{1.6}$$

We follow the same principle used in the papers [3, 4]. In this paper, we find all the eta quotients in $M_1(\Gamma_0(20), \chi_1)$ and $M_1(\Gamma_0(20), \chi_2)$, and we determine their Fourier coefficients.

2. Preliminary results

Throughout the remainder of this paper we use the notation $q = q(z) = e^{2\pi i z}$ with $z \in \mathbb{H}$. We define the Eisenstein series $E_{\chi_1,\chi_0}(q)$, $E_{\chi_2,\chi_0}(q)$ and $E_{\chi_2,\chi_1}(q)$ by

$$E_{\chi_1,\chi_0}(q) = \sum_{n=1}^{\infty} \sigma_{(\chi_1,\chi_0)}(n) q^n, \quad E_{\chi_2,\chi_0}(q) = 1 + \sum_{n=1}^{\infty} \sigma_{(\chi_2,\chi_0)}(n) q^n,$$

$$E_{\chi_3,\chi_1}(q) = \sum_{n=1}^{\infty} \sigma_{(\chi_3,\chi_1)}(n) q^n.$$

In view of (2.1) for $N = 20$, we define an eta quotient $f(z)$ by

$$f(z) = \eta^{r_1}(z)\eta^{r_2}(2z)\eta^{r_4}(4z)\eta^{r_5}(5z)\eta^{r_{10}}(10z)\eta^{r_{20}}(20z). \tag{2.1}$$

Theorem 2.1. *Let $f(z) \in M_1(\Gamma_0(20), \chi_1)$ be an eta quotient given by (2.1), and let $f(z) = \sum_{n=0}^{\infty} a_n q^n$ be its Fourier series expansion. Then*

$$f(z) = b_1 E_{\chi_1,\chi_0}(q) + b_2 E_{\chi_1,\chi_0}(q^5)$$

for unique scalars $b_1, b_2 \in \mathbb{C}$, and the Fourier coefficients a_n are given by

$$a_n = b_1 \sigma_{\chi_1,\chi_0}(n) + b_2 \sigma_{\chi_1,\chi_0}(n/5) \text{ for } n \ge 1.$$

Proof: It follows from (1.6) and [1, Theorem 5.9] that the set of Eisenstein series $\{E_{\chi_1,\chi_0}(q),\ E_{\chi_1,\chi_0}(q^5)\}$ is a basis for $M_1(\Gamma_0(20), \chi_1)$. Thus,

$$f(z) = b_1 E_{\chi_1,\chi_0}(q) + b_2 E_{\chi_1,\chi_0}(q^5)$$

for some unique scalars $b_1, b_2, \in \mathbb{C}$, from which the assertion follows by equating the coefficients of q^n on both sides. □

Similarly to Theorem 2.1, we prove the following theorem.

Theorem 2.2. *Let* $f(z) \in M_1(\Gamma_0(20), \chi_2)$ *be an eta quotient given by* (2.1), *and let* $f(z) = \sum_{n=0}^{\infty} a_n q^n$ *be its Fourier series expansion. Then*

$$f(z) = b_1 E_{\chi_2,\chi_0}(q) + b_2 E_{\chi_3,\chi_1}(q)$$

for unique scalars $b_1, b_2 \in \mathbb{C}$, *and the Fourier coefficients* a_n *are given by*

$$a_n = b_1 \sigma_{\chi_2,\chi_0}(n) + b_2 \sigma_{\chi_3,\chi_1}(n) \text{ for } n \geq 1,$$
$$a_0 = b_1.$$

We use the following lemma to determine if certain eta quotients are modular forms. See [5, Theorem 5.7, p. 99], [6, Corollary 2.3, p. 37], [7, p. 174], and [8].

Lemma 2.3. *Let* $f(z)$ *be an eta quotient given by* (2.1), *and let* $k = \frac{1}{2}\sum_{1\leq\delta|N} r_\delta$ *and* $s = \prod_{1\leq\delta|N} \delta^{r_\delta}$. *Suppose that the following conditions are satisfied:*

(i) $\sum_{1\leq\delta|N} \delta \cdot r_\delta \equiv 0\,(mod\ 24)$,

(ii) $\sum_{1\leq\delta|N} \frac{N}{\delta} \cdot r_\delta \equiv 0\,(mod\ 24)$,

(iii) $\sum_{1\leq\delta|N} \frac{\gcd(d,\delta)^2 \cdot r_\delta}{\delta} \geq 0$ *for each positive divisor* d *of* N,

(iv) k *is an integer.*

Then $f(z) \in M_k(\Gamma_0(N), \chi)$, *where the character* χ *is given by*

$$\chi(m) = \left(\frac{(-1)^k s}{m}\right).$$

We utilize the following notation for eta-quotients

$$[\delta_1, r_{\delta_1}; \delta_2, r_{\delta_2}; \ldots, \delta_{d-1}, r_{\delta_{d-1}}; \delta_d, r_{\delta_d}] = \prod_{i=1}^{d} \eta^{r_{\delta_i}}(\delta_i z),$$

We take $N = 20$ and $k = 1$ in Lemma 2.3 to obtain the following theorem.

Theorem 2.4. *Let* $f(z)$ *be an eta quotient given by* (2.1), *which satisfies the conditions (i)-(iv) in Lemma 2.3 with*

$$r_1 + r_2 + r_4 + r_5 + r_{10} + r_{20} = 2. \tag{2.2}$$

Then $f(z) \in M_1(\Gamma_0(20), \chi)$, *where the character* χ *is determined by*

$$f(z) \in \begin{cases} M_1(\Gamma_0(20), \chi_1), & \text{if } r_4 + r_5 + r_{20} \equiv 0 \pmod 2; \\ M_1(\Gamma_0(20), \chi_2), & \text{if } r_2 + r_5 + r_{20} \equiv 0 \pmod 2. \end{cases} \tag{2.3}$$

Proof: For $N = 20$, we have

$$s = \prod_{1\leq\delta|20} \delta^{r_\delta} = 1^{r_1} 2^{r_2} 4^{r_4} 5^{r_5} 10^{r_{10}} 20^{r_{20}} = 2^{r_2+2r_4+r_{10}+2r_{20}} 5^{r_5+r_{10}+r_{20}}.$$

The conditions (i) and (ii) in Lemma 2.3 becomes

$$r_1 + 2r_2 + 4r_4 + 5r_5 + 10r_{10} + 20r_{20} \equiv 0 \pmod{24},$$

$$20r_1 + 10r_2 + 5r_4 + 4r_5 + 2r_{10} + r_{20} \equiv 0 \pmod{24}.$$

Then,

$$r_2 + r_{10} \equiv 0 \pmod 2.$$

The character associated to $f(z)$ is given by

$$\chi(m) = \left(\frac{(-1)^k s}{m}\right) = \left(\frac{-s}{m}\right).$$

Hence,

$$\begin{aligned}\chi &= \chi_1 \quad \text{if } r_4 + r_5 + r_{20} \equiv 0 \pmod 2,\\ \chi &= \chi_2 \quad \text{if } r_5 + r_{10} + r_{20} \equiv 0 \pmod 2 \quad \text{and } r_2 + r_{10} \equiv 0 \pmod 2.\end{aligned}$$

These match the given definitions of χ_1 and χ_2, and the result follows. □

3. Main results

Theorem 3.1. *Let $f(z)$ be an eta quotient given by (2.1). Then we have $f(z) \in M_1(\Gamma_0(20), \chi_1)$ if and only if*

$$\begin{aligned} r_1 + 2r_2 + 4r_4 + 5r_5 + 10r_{10} + 20r_{20} &\equiv 0 \pmod{24},\\ 20r_1 + 10r_2 + 5r_4 + 4r_5 + 2r_{10} + r_{20} &\equiv 0 \pmod{24},\\ 0 \le v_{1/c}(f) < 2 \text{ for } c &= \frac{1}{20}, \frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}, \frac{1}{1},\\ r_1 + r_2 + r_4 + r_5 + r_{10} + r_{20} &= 2,\\ r_4 + r_5 + r_{20} &\equiv 0 \pmod 2. \end{aligned}$$

Proof: Let $f(z) \in M_1(\Gamma_0(20), \chi_1)$ be an eta quotient given by (2.1). By (1.6) we have $\dim(M_1(\Gamma_0(20), \chi_1)) = 2$. We define the eta quotients $f_1(z), f_2(z)$ by

$$\begin{aligned} f_1(z) &= [-1, 1; 2, 2; 4, -1; 5, 1; 10, 0; 20, 1],\\ f_2(z) &= [1, 1; 2, 0; 4, 1; 5, -1; 10, 2; 20, -1]. \end{aligned}$$

By Lemma 2.3, we have $f_1(z), f_2(z) \in M_1(\Gamma_0(20), \chi_1)$. One can easily see that the set $\{f_1(z), f_2(z)\}$ is linearly independent, and so it is a basis for $M_1(\Gamma_0(20), \chi_1)$. Appealing to (1.4) and (1.5), we have

$v_{1/c}(\cdot)/c$	1	1/2	1/4	1/5	1/10	1/20
f_1	0	1/2	0	1	1/2	1
f_2	1	1/2	1	0	1/2	0

Thus, for any $b_1, b_2 \in \mathbb{C}$ we have

$$v_{1/c}(b_1 f_1 + b_2 f_2 \in \mathbb{N}_0.$$

As $f(z)$ can be expressed as a linear combination of $f_1(z)$ and $f_2(z)$, we have

$$v_{1/c}(f) \in \mathbb{N}_0,$$

from which the second and first assertions follow, respectively. The third assertion follows from [6, Corollary 2.3] and the fifth assertion follows from (2.3). The converse follows from Theorem 2. □

Theorem 3.2. *Let $f(z)$ be an eta quotient given by (2.1). Then we have $f(z) \in M_1(\Gamma_0(20), \chi_2)$ if and only if*

$$\begin{aligned} r_1 + 2r_2 + 4r_4 + 5r_5 + 10r_{10} + 20r_{20} &\equiv 0 \pmod{24},\\ 20r_1 + 10r_2 + 5r_4 + 4r_5 + 2r_{10} + r_{20} &\equiv 0 \pmod{24},\\ 0 \le v_{1/c}(f) < 2 \text{ for } c &= 1, 2, 4, 5, 10, 20,\\ r_1 + r_2 + r_4 + r_5 + r_{10} + r_{20} &= 2,\\ r_2 + r_5 + r_{20} &\equiv 0 \pmod 2. \end{aligned}$$

Proof: Let $f(z) \in M_1(\Gamma_0(20), \chi_2)$ be an eta quotient given by (2.1). By (1.6) we have $\dim(M_1(\Gamma_0(20), \chi_2)) = 4$. We define the eta quotients $f_1(z)$ and $f_2(z)$ by

$$f_1(z) = [1, -1; 2, 1; 4, 1; 5, 1; 10, 1; 20, -1],$$
$$f_2(z) = [1, 1; 2, 1; 4, -1; 5, -1; 10, 1; 20, 1].$$

By Lemma 2.3, we have $f_1(z), f_2(z) \in M_1(\Gamma_0(20), \chi_2)$. One can easily see that the set $\{f_1(z), f_2(z)\}$ is a basis for $M_1(\Gamma_0(20), \chi_2)$. By (1.4) and (1.5), we have

$v_{1/c}(\cdot)/c$	1	1/2	1/4	1/5	1/10	1/20
f_1	0	1/2	1	1	1/2	0
f_2	1	1/2	0	0	1/2	1

Then, $v_{1/c}(b_1 f_1 + b_2 f_2 \in \mathbb{N}_0$ for any $b_1, b_2 \in \mathbb{C}$. As $f(z)$ can be expressed as a linear combination of $f_1(z)$ and $f_2(z)$, we have $v_{1/c}(f) \in \mathbb{N}_0$, from which the second and first assertions follow, respectively. The third assertion follows from [6, Corollary 2.3] and the fifth assertion follows from (2.3). The converse follows from Theorem 2. □

There are 4 eta quotients in $M_1(\Gamma_0(20), \chi_1)$ and 3 eta quotients in $M_1(\Gamma_0(20), \chi_2)$. We found all the eta quotients with Sagemath [9] using Theorems 3.1 and 3.2. We then determined their Fourier coefficients using Theorems 2.1 and 2.2. All these eta quotients and their Fourier coefficients are listed in the appendices (Table 1 and 2).

4. Applications

Theorem 4.1. *Let $f(z) \in M_1(\Gamma_0(20), \chi_1)$ with the Fourier series representation*

$$f(z) = \sum_{n=0}^{\infty} a_n q^n.$$

Then For all $n \geq 0$, we have $a_n = 0$ if $n \equiv 3, 7, 11, 15, \pmod{20}$.

Proof: Suppose $n \equiv 3, 7, 11, 15, \pmod{20}$. Then

$$\chi_1(n) = \left(\frac{-4}{n}\right) = \left(\frac{n}{4}\right) = -1.$$

Also, for all positive divisors d of n, we have

$$\chi_1(n/d) = \left(\frac{-4}{n/d}\right) = \left(\frac{-4}{nd}\right) = \left(\frac{-4}{n}\right)\left(\frac{-4}{d}\right) = -\left(\frac{-4}{d}\right) = -\chi_1(d).$$

By pairing $\chi_1(d)$ and $\chi_1(n/d)$ for all $d \mid n$, we obtain

$$\sum_{d|n} \chi_1(d) = 0.$$

The assertion now follows from (1.4), (1.3), and Theorem 2. □

Theorem 4.2. *Let $f(z) \in M_1(\Gamma_0(20), \chi_2)$ with the Fourier series representation*

$$f(z) = \sum_{n=0}^{\infty} a(n) q^n.$$

Then for all $n \geq 0$, we have $a(n) \equiv \pmod{\sigma_{(\chi_3, \chi_1)}(n)}$ if $n \equiv 11, 13, 17, 19 \pmod{20}$.

Proof: Suppose $n \equiv 11, 13, 17, 19 \pmod{20}$. Then

$$\chi_2(n) = \left(\frac{-20}{n}\right) = \left(\frac{n}{20}\right) = -1.$$

Also, for all positive divisors d of n, we have

$$\chi_2(n/d) = \left(\frac{-20}{n/d}\right) = \left(\frac{-20}{nd}\right) = \left(\frac{-20}{n}\right)\left(\frac{-20}{d}\right) = -\left(\frac{-20}{d}\right) = -\chi_2(d).$$

By pairing $\chi_2(d)$ and $\chi_2(n/d)$ for all $d \mid n$, we obtain

$$\sum_{d|n} \chi_2(d) = 0. \tag{4.1}$$

The assertion now follows from (4.1), (1.3), and Theorem 2.2. □

The following corollary follows immediately from Theorem 4.1 and 4.2.

Corollary 4.3. *If an eta quotient $f(z)$ given by (2.1) is a modular form of weight 1 with the Fourier series representation $f(z) = \sum_{n=0}^{\infty} a_n q^n$, then for all $n \geq 0$ we have*

$$a_n \equiv \begin{cases} 0 \pmod{\sigma_{\chi_3,\chi_1}(n)}, & \text{if } n \equiv 3, 7, 11, 15 \pmod{20},\ r_4 + r_5 + r_{20} \equiv 0 \pmod{2}; \\ 0 \pmod{\sigma_{\chi_3,\chi_1}(n)}, & \text{if } n \equiv 11, 13, 17, 19 \pmod{20},\ r_2 + r_5 + r_{20} \equiv 0 \pmod{2}. \end{cases}$$

5. Conclusions

In this paper, we have systematically investigated eta quotients of level 20 and weight 1 in the spaces $M_1(\Gamma_0(20), \chi_1)$ and $M_1(\Gamma_0(20), \chi_2)$, where χ_1 and χ_2 are Dirichlet characters modulo 20 associated with the Legendre-Jacobi-Kronecker symbols $\left(\frac{-4}{\cdot}\right)$ and $\left(\frac{-20}{\cdot}\right)$, respectively. Our main contributions include:

(1) Classification of Eta Quotients:
Using the conditions derived from Lemma 2.3, we identified all possible eta quotients in $M_1(\Gamma_0(20), \chi_1)$ and $M_1(\Gamma_0(20), \chi_2)$. Specifically, we found that there are 4 eta quotients in $M_1(\Gamma_0(20), \chi_1)$ and 3 in $M_1(\Gamma_0(20), \chi_2)$, as listed in the appendices (Tables 1 and 2).

(2) Fourier Coefficients:
By expressing these eta quotients in terms of Eisenstein series (Theorems 2.1 and 2.2), we derived explicit formulas for their Fourier coefficients. These results provide a concrete way to compute the coefficients a_n for each modular form.

(3) Vanishing and Congruence Properties: We established vanishing conditions for the Fourier coefficients a_n in certain arithmetic progressions (Theorems 4.1 and 4.2). Specifically, we showed that:

   (a) For forms in $M_1(\Gamma_0(20), \chi_1)$, $a_n = 0$ when $n \equiv 3, 7, 11, 15 \pmod{20}$.

   (b) For forms in $M_1(\Gamma_0(20), \chi_2)$, $a_n \equiv 0 \pmod{\sigma_{(\chi_3,\chi_1)}(n)}$ when $n \equiv 11, 13, 17, 19 \pmod{20}$.

These findings contribute to the broader understanding of modular forms and eta quotients, particularly in cases where the weight is odd and the level is composite. The results may also have applications in number theory, such as in the study of partition functions, quadratic forms, and L-functions.

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Table 1. Eta quotients in $M_1(\Gamma_0(20), \chi_1)$ and their Fourier coefficients

No.	r_1	r_2	r_4	r_5	r_{10}	r_{20}	(b_1, b_2)
1	-4	10	-4	0	0	0	$(4, 0)$
2	-1	2	-1	1	0	1	$(1, 0)$
3	0	0	0	-4	10	-4	$(0, 2)$
4	1	0	1	-1	2	-1	$(-1, 0)$

Table 2. Eta quotients in $M_1(\Gamma_0(20), \chi_2)$ and their Fourier coefficients

No.	r_1	r_2	r_4	r_5	r_{10}	r_{20}	(b_1, b_2)
1	-2	5	-2	-2	5	-2	$(1, 1)$
2	-1	1	1	1	1	-1	$(1, 0)$
3	1	1	-1	-1	1	1	$(0, 1)$

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