

New fixed point theorems for nonlinear contractive mappings of integral type involving ω -distance and its applications

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Abstract. This paper introduces a new class of contractive mappings of integral type involving the concept of ω -distance in complete metric spaces. We establish a fixed point theorem for these mappings, providing sufficient conditions for the existence and uniqueness of fixed points. We also provide an example to illustrate the validity of our results and discuss applications of this fixed point theorem in solving integral equations, demonstrating its practical relevance.

Keywords: Fixed point theorem; Integral-type contraction; ω -distance; Complete metric space; Integral equation

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1. INTRODUCTION

Fixed point theory is a cornerstone of nonlinear analysis, providing powerful tools for solving a wide range of problems in mathematics, engineering, and other disciplines. Its central concept revolves around finding solutions to equations of the form $T(x) = x$, where T is a mapping from a set to itself, and x is a fixed point of T .

The Banach contraction principle, established by S. Banach in 1922, is one of the fundamental results in fixed point theory. It states that a contraction mapping on a complete metric space has a unique fixed point. It is well known that every Banach contraction mapping is continuous; however, Kannan [1] showed that a discontinuous function satisfying their contractive condition has a unique fixed point. Among these generalizations, we must quote the fixed point results due to Prasad [2] wherein the respective author proved an analogous version of fixed point theorem due to Kannan via conditional contractive mappings and some other relational metrical notions. The Banach contraction principle has been extensively generalized and extended in various directions. Ali et al. [3] generalized classical Banach contraction principle in the setup of quasi-metric spaces. They introduced Suzuki-type contractions of quasi-metric spaces and proved some fixed point results. In [4] a family of hybrid contractions, termed Jaggi-type hybrid $(\aleph - \tau)$ -contractive mapping is proposed in metric space equipped with a graph and new conditions under which the mapping is a Picard operator are studied and also in [5] the authors established the existence and uniqueness of a common coupled fixed point result in the framework of complete symmetric S_b -metric space.

On another point of note, recently in [6] the author prove relational analogue of the Banach contraction principle in the settings of Menger probabilistic metric spaces under a Hadžić-type t -norm. Also, in [7] the authors proved an analogous version of fixed point theorem due to Suzuki using R -sequential limit property and some other relation theoretic metrical variants. For other related results, we refer to the references [8, 9, 10, 11, 12, 13].

One such direction involves generalizing the notion of contractive mappings. In particular, integral-type contractions, initiated by Branciari [11], have gained considerable attention. These contractions replace the usual metrical inequality with an integral inequality, offering a more general framework. Branciari's work sparked further research, leading to numerous fixed point theorems involving integral-type contractions under various conditions (see [14, 15, 16, 17]).

Another important generalization involves weakening the metric itself. The concept of ω -distance, introduced by Kada, Suzuki, and Takahashi [18], replaces the metric with a function satisfying weaker properties, such as lacking symmetry or the triangle inequality. This allows for the study of fixed points in spaces where the standard metric assumptions are not satisfied. Recently, in [19] the author established fixed point theorems for generalized nonlinear contractive mappings using the concept of ω -distance on metric spaces endowed with an arbitrary binary relation. We refer to some notable

references discussing fixed point theorems for nonlinear contractive mappings involving ω -distance [20, 21, 22].

The combination of integral-type contractions and ω -distance provides a powerful framework for studying fixed points in a more general setting. This paper aims to contribute to this area by introducing a new class of contractive mappings of integral type involving ω -distance and establishing corresponding fixed point theorems. We also explore applications of these theorems in solving integral equations.

2. PRELIMINARIES

In this section, we present some basic definitions and results that are necessary for our discussion.

Definition 2.1. Let (X, d) be a metric space. A function $\omega : X \times X \rightarrow [0, \infty)$ is called an ω -distance on X if the following conditions hold:

- (i) $\omega(x, z) \leq \omega(x, y) + \omega(y, z)$, for all $x, y, z \in X$;
- (ii) $\omega(\cdot, y) : X \rightarrow [0, \infty)$ is lower semicontinuous for all $y \in X$; that is, if $\{x_n\}$ converges to x , then $\omega(x, y) \leq \liminf_{n \rightarrow \infty} \omega(x_n, y)$;
- (iii) For any $\epsilon > 0$, there exists $\delta > 0$ such that $\omega(z, x) \leq \delta$ and $\omega(z, y) \leq \delta$ imply $d(x, y) \leq \epsilon$, for all $x, y, z \in X$.

Example 2.1. The metric d itself is an ω -distance.

Example 2.2. Let $X = \mathbb{R}$ and $d(x, y) = |x - y|$. Define $\omega(x, y) = \max\{y - x, 0\}$. Then ω is an ω -distance on X .

Definition 2.2. Let Ψ be the family of mappings $\psi : [0, \infty) \rightarrow [0, \infty)$ which is Lebesgue integrable on each compact subset of $[0, \infty)$ such that for each $\epsilon > 0$, $\int_0^\epsilon \psi(t)dt > 0$.

Definition 2.3. Let (X, d) be a metric space and $T : X \rightarrow X$. T is called a *Branciari contraction* if there exists $k \in (0, 1)$ such that for all $x, y \in X$,

$$\int_0^{d(Tx, Ty)} \psi(t)dt \leq k \int_0^{d(x, y)} \psi(t)dt,$$

where $\psi \in \Psi$.

Lemma 2.1. Let X be a metric space with metric d and let ω be an ω -distance on X . Let $\{x_n\}$ and $\{y_n\}$ be sequences in X , let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $\mathbb{R}^+$ converging to 0, and let $x, y, z \in X$. Then the following hold:

- (i) If $\omega(x_n, y) \leq \alpha_n$ and $\omega(x_n, z) \leq \beta_n$ for any $n \in \mathbb{N}$, then $y = z$. In particular, if $\omega(x, y) = 0$ and $\omega(x, z) = 0$, then $y = z$;
- (ii) If $\omega(x_n, y_n) \leq \alpha_n$ and $\omega(x_n, z_n) \leq \beta_n$ for any $n \in \mathbb{N}$, then $\{y_n\}$ converges to z ;
- (iii) If $\omega(x_n, x_m) \leq \alpha_n$ for any $n, m \in \mathbb{N}$ with $n > m$, then $\{x_n\}$ is a Cauchy sequence;
- (iv) If $\omega(x, x_n) \leq \alpha_n$ for any $n \in \mathbb{N}$, then $\{x_n\}$ is a Cauchy sequence.

3. MAIN RESULTS

Now, we introduce our new class of contractive mappings and establish the corresponding fixed point theorems.

Definition 3.1. Let (X, d) be a metric space and ω be an ω -distance on X . Let $T : X \rightarrow X$ be a mapping. We say that T is a *generalized ω -integral type contraction* if there exists a function $\psi \in \Psi$, and there exists $k \in (0, 1)$ such that for all $x, y \in X$,

$$\int_0^{\omega(Tx, Ty)} \psi(t) dt \leq k \int_0^{M(x, y)} \psi(t) dt,$$

where

$$M(x, y) = \max \left\{ \omega(x, y), \omega(x, Tx), \omega(y, Ty), \frac{\omega(x, Ty) + \omega(y, Tx)}{2} \right\}.$$

Theorem 3.1. Let (X, d) be a complete metric space and ω be an ω -distance on X . Let $T : X \rightarrow X$ be a generalized ω -integral type contraction. Then T has a unique fixed point in X .

Proof: Let $x_0 \in X$ be arbitrary. Define a sequence $\{x_n\}$ in X by $x_{n+1} = T(x_n)$ for all $n \geq 0$. Consider $\omega(x_{n+1}, x_n) = \omega(Tx_n, Tx_{n-1})$. By the definition of T , we have

$$\int_0^{\omega(Tx_{n-1}, Tx_n)} \psi(t) dt \leq k \int_0^{M(x_{n-1}, x_n)} \psi(t) dt,$$

where

$$\begin{aligned} M(x_{n-1}, x_n) &= \max \left\{ \omega(x_{n-1}, x_n), \omega(x_{n-1}, Tx_{n-1}), \omega(x_n, Tx_n), \frac{\omega(x_{n-1}, Tx_n) + \omega(x_n, Tx_{n-1})}{2} \right\} \\ &= \max \left\{ \omega(x_{n-1}, x_n), \omega(x_{n-1}, x_n), \omega(x_n, x_{n+1}), \frac{\omega(x_{n-1}, x_{n+1}) + \omega(x_n, x_n)}{2} \right\} \\ &= \max \left\{ \omega(x_{n-1}, x_n), \omega(x_n, x_{n+1}), \frac{\omega(x_{n-1}, x_{n+1}) + 0}{2} \right\} \\ &\leq \max \left\{ \omega(x_{n-1}, x_n), \omega(x_n, x_{n+1}), \frac{\omega(x_{n-1}, x_n) + \omega(x_n, x_{n+1})}{2} \right\}. \end{aligned}$$

Let $a_n = \omega(x_n, x_{n+1})$. Then,

$$\int_0^{a_n} \psi(t) dt \leq k \int_0^{\max\{a_{n-1}, a_n, \frac{a_{n-1} + a_n}{2}\}} \psi(t) dt. \quad (3.1)$$

If $a_n \leq a_{n-1}$ for all $n \geq N$, then $\max\{a_{n-1}, a_n\} = a_{n-1}$ and

$$\int_0^{a_n} \psi(t) dt \leq k \int_0^{a_{n-1}} \psi(t) dt.$$

Let $I_n = \int_0^{a_n} \psi(t) dt$. Then $I_n \leq k I_{n-1}$. This implies $I_n \rightarrow 0$. Since $\psi \geq 0$ and $\int_0^r \psi(t) dt$ is strictly increasing, then $a_n \rightarrow 0$.

On the other hand, if $a_n > a_{n-1}$ for infinitely many n , let n be such that $a_n > a_{n-1}$. Then $\max\{a_{n-1}, a_n\} = a_n$ and

$$\int_0^{a_n} \psi(t) dt \leq k \int_0^{a_n} \psi(t) dt.$$

Hence

$$(1 - k) \int_0^{a_n} \psi(t) dt \leq 0.$$

Since $1 - k > 0$ and $\int_0^{a_n} \psi(t) dt \geq 0$, then $\int_0^{a_n} \psi(t) dt = 0$. Since $\psi \geq 0$, this means $a_n = 0$. But we assumed $a_n > a_{n-1} \geq 0$. So this is a contradiction unless $a_{n-1} = 0$ and $a_n = 0$. Thus, we cannot have $a_n > a_{n-1}$ with $a_n > 0$.

Therefore, it must be that $a_n \leq a_{n-1}$ for all n sufficiently large. This implies that

$$\lim_{n \rightarrow \infty} \omega(x_n, x_{n+1}) = 0.$$

Now, we show that $\{x_n\}$ is a Cauchy sequence. Since $\{a_i\}$ is non-increasing and converges to 0, the sum $\sum_{i=n}^{\infty} a_i$ converges. Let $S_n = \sum_{i=n}^{\infty} a_i$. Then $S_n \rightarrow 0$ as $n \rightarrow \infty$. For $m > n$, we have

$$\begin{aligned}\omega(x_n, x_m) &\leq \omega(x_n, x_{n+1}) + \omega(x_{n+1}, x_{n+2}) + \cdots + \omega(x_{m-1}, x_m) \\ &= a_n + a_{n+1} + \cdots + a_{m-1} \leq \sum_{i=n}^{\infty} \omega(x_i, x_{i+1}) = S_n.\end{aligned}$$

Since $S_n \rightarrow 0$ as $n \rightarrow \infty$, then

$$\lim_{n, m \rightarrow \infty} \omega(x_n, x_m) = 0. \quad (3.2)$$

Let $\epsilon > 0$ and δ denote the number in the property (iii) of ω -distance. It follows from (3.2) that there exists $N \in \mathbb{N}$ satisfying

$$\omega(x_N, x_n) < \delta \quad \text{and} \quad \omega(x_N, x_m) < \delta, \quad \forall m, n \geq N,$$

which together with the property (iii) of ω -distance yields that

$$d(x_n, x_m) < \epsilon, \quad \forall m, n \geq N.$$

This shows that $\{x_n\}$ is a Cauchy sequence. Since (X, d) is a complete metric space, $\{x_n\}$ converges to some $x \in X$. That is,

$$\lim_{n \rightarrow \infty} x_n = x.$$

Now, we prove that x is a fixed point of T . Observe that (3.2) guarantees that for each $\epsilon > 0$, there exists $N_\epsilon \in \mathbb{N}$ satisfying

$$\omega(x_n, x_m) < \epsilon, \quad \forall m, n \geq N_\epsilon,$$

which together with the property (iii) of ω -distance and $\lim_{n \rightarrow \infty} x_n = x$ yields that

$$0 \leq \omega(x_n, x) \leq \liminf_{n \rightarrow \infty} \omega(x_n, x_m) \leq \epsilon, \quad \forall n \geq N_\epsilon,$$

which gives that

$$\lim_{n \rightarrow \infty} \omega(x_n, x) = 0. \quad (3.3)$$

Also, since $\omega(\cdot, y)$ is lower semi-continuous, we have

$$0 \leq \omega(x, x) \leq \liminf_{n \rightarrow \infty} \omega(x_n, x) = 0.$$

Hence, $\omega(x, x) = 0$. On the other hand,

$$\omega(x, Tx) \leq \liminf_{n \rightarrow \infty} \omega(x_n, Tx) = \liminf_{n \rightarrow \infty} \omega(Tx_{n-1}, Tx) \leq \liminf_{n \rightarrow \infty} M(x_{n-1}, x).$$

It is easy to see that $M(x_{n-1}, x) \rightarrow \omega(x, Tx)$ as $n \rightarrow \infty$. Since $k \in (0, 1)$, then

$$\omega(x, Tx) = 0.$$

Now, since $\omega(x, Tx) = 0$ and $\omega(x, x) = 0$, in view of Lemma 2.1 we deduce that $Tx = x$ and x is a fixed point of T .

To prove uniqueness, suppose y is another fixed point of T , i.e., $Ty = y$ and $x \neq y$. Then,

$$\begin{aligned}M(x, y) &= \max \left\{ \omega(x, y), \omega(x, Tx), \omega(y, Ty), \frac{\omega(x, Ty) + \omega(y, Tx)}{2} \right\} \\ &= \max \left\{ \omega(x, y), \omega(x, x), \omega(y, y), \frac{\omega(x, y) + \omega(y, x)}{2} \right\} \\ &= \max \left\{ \omega(x, y), \frac{\omega(x, y) + \omega(y, x)}{2} \right\}.\end{aligned}$$

Now, we consider the following two cases:

Case 1. If $\omega(y, x) \leq \frac{\omega(x, y) + \omega(y, x)}{2} \leq \omega(x, y)$, then $M(x, y) = \omega(x, y)$. We have

$$\int_0^{\omega(x, y)} \psi(t) dt = \int_0^{\omega(Tx, Ty)} \psi(t) dt \leq k \int_0^{M(x, y)} \psi(t) dt = k \int_0^{\omega(x, y)} \psi(t) dt < \int_0^{\omega(x, y)} \psi(t) dt,$$

which is a contradiction.

Case 2. If $\omega(x, y) \leq \frac{\omega(x, y) + \omega(y, x)}{2} < \omega(y, x)$, then $M(y, x) = \omega(y, x)$. We have

$$\begin{aligned} \int_0^{\omega(x, y)} \psi(t) dt &< \int_0^{\omega(y, x)} \psi(t) dt = \int_0^{\omega(Ty, Tx)} \psi(t) dt \leq k \int_0^{M(y, x)} \psi(t) dt \\ &= k \int_0^{\omega(y, x)} \psi(t) dt < \int_0^{\omega(y, x)} \psi(t) dt, \end{aligned}$$

which is a contradiction. Hence, in two cases, $\omega(x, y) = 0$, which together with $\omega(x, x) = 0$ and Lemma 2.1 implies that $y = x$. This completes the proof. $\square$

Example 3.1. Let $X = [0, 1]$ with the usual metric $d(x, y) = |x - y|$. Define $\omega(x, y) = \max\{y - x, 0\}$. Then ω is an ω -distance on X . Define $T : X \rightarrow X$ by $T(x) = x/4$ for all $x \in X$. Let $\psi(t) = 1$ for all $t \in [0, \infty)$. Then ψ is a Lebesgue integrable function. We need to show that

$$\int_0^{\omega(Tx, Ty)} \psi(t) dt \leq k \int_0^{M(x, y)} \psi(t) dt$$

for some $k \in (0, 1)$. We have

$$\omega(Tx, Ty) = \max\{Ty - Tx, 0\} = \max\{y/4 - x/4, 0\} = \frac{1}{4} \max\{y - x, 0\} = \frac{1}{4} \omega(x, y),$$

and

$$\begin{aligned} M(x, y) &= \max \left\{ \omega(x, y), \omega(x, Tx), \omega(y, Ty), \frac{\omega(x, Ty) + \omega(y, Tx)}{2} \right\} \\ &= \max \left\{ \max\{y - x, 0\}, \max\{-\frac{3}{4}x, 0\}, \max\{-\frac{3}{4}y, 0\}, \frac{\max\{\frac{y}{4} - x, 0\} + \max\{\frac{x}{4} - y, 0\}}{2} \right\}. \end{aligned}$$

If $y > x$, then $M(x, y) = y - x$. Hence,

$$\int_0^{\omega(Tx, Ty)} \psi(t) dt = \frac{1}{4} \omega(x, y) = \frac{1}{4} M(x, y) = \frac{1}{4} \int_0^{M(x, y)} \psi(t) dt.$$

Thus, T satisfies the generalized ω -integral type contraction with $k = 1/4$. Therefore, by Theorem 3.1, T has a unique fixed point in X .

4. APPLICATION TO INTEGRAL EQUATIONS

In this section, we apply our fixed point theorem to prove the existence and uniqueness of solutions to a class of nonlinear integral equations.

Integral equations represent a fundamental class of mathematical problems that have found widespread use in fields ranging from physics and engineering to economics and biology. They offer an alternative formulation to differential equations for many physical phenomena and often provide a more natural and efficient way to model certain problems.

Consider the following integral equation:

$$x(t) = \int_0^\tau K(t, s, x(s)) ds, \quad \text{for } t \in [0, \tau],$$

where $K : [0, \tau] \times [0, \tau] \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function.

Let $X = C([0, \tau], \mathbb{R})$ be the space of continuous real-valued functions on $[0, \tau]$, equipped with the metric

$$d(x, y) = \max_{t \in [0, \tau]} |x(t) - y(t)|.$$

Then (X, d) is a complete metric space. Define an operator $T : X \rightarrow X$ by

$$T(x)(t) = \int_0^\tau K(t, s, x(s)) ds.$$

It is easy to see that a solution to this integral equation is a fixed point of the operator T .

Assume that there exists $c \in (0, 1)$ such that for all $t, s \in [0, \tau]$ and $u, v \in \mathbb{R}$,

$$|K(t, s, u) - K(t, s, v)| \leq \frac{c}{\tau} |u - v|.$$

We will show that T is a generalized ω -integral type contraction with $\omega(x, y) = d(x, y)$ and $\psi(t) = \frac{1}{2}t$. Then,

$$\begin{aligned} d(Tx, Ty) &= \max_{t \in [0, \tau]} |T(x)(t) - T(y)(t)| = \max_{t \in [0, \tau]} \left| \int_0^\tau K(t, s, x(s)) - K(t, s, y(s)) ds \right| \\ &\leq \max_{t \in [0, \tau]} \int_0^\tau |K(t, s, x(s)) - K(t, s, y(s))| ds \\ &\leq \max_{t \in [0, \tau]} \int_0^\tau \frac{c}{\tau} |x(s) - y(s)| ds = \frac{c}{\tau} \int_0^\tau d(x, y) ds = cd(x, y). \end{aligned}$$

Therefore,

$$\int_0^{d(Tx, Ty)} \frac{1}{2} t dt \leq \sqrt{c} \int_0^{d(x, y)} \frac{1}{2} t dt.$$

Since $d(x, y) = \omega(x, y)$, this shows that T is a generalized ω -integral type contraction with $\psi(t) = \frac{1}{2}t$ and $k = \sqrt{c} \in (0, 1)$.

By Theorem 3.1, T has a unique fixed point in X , which implies that the integral equation (4) has a unique solution in $C([0, \tau], \mathbb{R})$.

CONCLUSION

In this paper, we have introduced a new class of contractive mappings of integral type involving ω -distance and established several fixed point theorems for these mappings. This work opens up avenues for further research, including:

- Exploring different types of ω -distance and their impact on fixed point results;
- Investigating applications in other areas of mathematics and engineering, such as differential equations, optimization, and image processing;
- Developing numerical methods for approximating the fixed points of these mappings.

By combining the power of integral-type contractions and ω -distance, we can continue to expand the scope of fixed point theory and its applications.

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